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Performance Modeling and Estimation of Serial System Exhibiting k-out-of-n: G Scheme with Controller Failure

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Abstract

The k-out-of-n system is a widely used concept in various fields. The significance of this system lies in its ability to ensure reliability, safety, and continuity of operation in various settings, including home appliances, military equipment, communication networks, and manufacturing systems. The system consists of two subsystems, each of which employs the k-out-of-n: G scheme. Our aim is to estimate the performance of a system that implements a k-out-of-n: G scheme with a controller using Copula's properties. This system is susceptible to two types of failures; partial and complete failures. If the system fails partially, it is repaired using General repair techniques while completely failed state is repaired using Copula repair techniques. The system is investigated using supplementary variable techniques and the Laplace transformation to obtain explicit expressions for availability, reliability, mean time to failure (MTTF), sensitivity, and expected profit functions, all of which are numerically validated. The outcomes/results are displayed in tables and figures, enabling us to draw the appropriate conclusions and offer recommendations. On the basis of numerical results, the conclusions were drawn. The findings of this study will aid maintenance managers in determining the appropriate time and technique for a system's maintenance.

Keywords: Availability, Controller, Exhibiting, Performance, Reliability, System, Scheme.

1. Introduction

Complex systems with numerous components and subsystems are prevalent nowadays. To reliably assess the performance of such systems, it is necessary to understand the nature of each component failure and how they interact. This will aid in the efficiency analysis of the system. In reality, a k-out-of-n system constitutes one of the most important systems. A system having n components can be described as a k-out-of-n system if and when k or more of the components operate. A relatively common sort of redundancy in fault-tolerant systems is the k-out-of-n system structure. It has several uses, such as in commercial systems, armed forces systems, transmission, engineering construction, transport, telecommunications and so on.

Numerous studies on the study of k-out-of-n systems have already been authored due to the wide range of practical applications. To name only a handful; Wu and Cui [23] investigated the dependability of a load-sharing k-out-of-n: G system with interrelated Markov subsystems. Zhao et al. [28] have given analysis of dependability of multifunctional-out-of-n: F a well-balanced system. Based on record values, Wang et al. [24], offered reliability evaluation of irreparable k-out-of-n: G systems with phased-mission requirements. Ahmadi [1], presented reliability and maintenance modeling for a load-sharing k-out-of-n system with concealed failures. Recently, Zhang et al. [12], provided maintenance analysis of a partial observable k-out-of-n system with load sharing units. A study on the reliability evaluation of a k-out-of-n: F system supported by a multi-state protective device in a shock environment was published in Xian et al. [26]. Dembinska et al [3] investigated the k-out-of-n system's reliability characteristics with a single cold standby unit. They primarily focused on the situation in which the system runs in discrete time. Hu et al. [8] modeled and assessed reliability for uncertain random cold standby k-out-of-m+n: G systems with uncertain parameters based on the system's uncertainty. Cekay [2] focused on the reliability of k-out-of-n systems that are created to carry out a certain task that consists of numerous separate phases. Eryilmaz and Devrim [3, 4] investigated reliability and ideal replacement strategy for a k-out-of-n system subject to

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random shocks. Gao et al. [5] suggested the ideal of an uncertain weighted k-out-of-n system and added an unknown variable to the weighted k-out-of-n system. A k-out-of-n system with multiple sorts of components with discrete independent lifetimes was presented with certain conditional reliability qualities by Jasinski [9]. For a repairable consecutive k-out-of-n: F system, Gokhan et al. [6], presented a time-dependent reliability analysis. An innovative mathematical model for reliability dealing with (k1, k2)-out-of-*n*: *G* systems consist of two different sorts of components was put forth by Wang et al. in [25]. The working times of two distinct are considered to be distributed exponentially with various factors. Zhang et al. [12] investigated reliability-based metrics and prognostic issues in a k-out-of-n system where each component's failure process depends only on its inherent qualities but also on the circumstances of its operational environment. Multi-performance weighted multi-state components are defined by Larsen et al. in [10], and two generalized multi-performance multi-state k-out-of-n system models are suggested. Pant et al. [13] investigated the cost rate and availability for a maintained k-out-of-n: G system that encounters numerous failure modes and undergoes routine inspection. In order to support decision-making in the preventive maintenance for unmanned underwater vehicles to monitor the condition of a subsea pipeline, Rykov et al. [15] demonstrated the feasibility of using a mathematical model of a k-out-of-nsystem. Rykov et al. [16] looked at the reliability properties of a k-out-of-n: F system in the scenario where a component failure increases the load on other components, lowering their residual lifetimes. Performance study of serial manufacturing lines with declining product quality was covered by Naebulharam and Zhang in [34]. Human-machine systems reliability: a series-parallel strategy for evaluation and improvement in the field of machine tools was presented by Rosa et al. in [35].

For all industrial systems, failure is inevitable and might take the form of partial or total failure. Any of these failures causes the system to operate less efficiently or to shut down entirely. Certain circumstances call for an immediate return to operation of a completely failed system. When this situation arises, we use the Copula approach. Copula technique is a powerful technique for describing interdependence among variables that has gotten a lot of interest in a lot of domains. The joint lifetime distribution can be generated by modeling component dependence using a Copula function, making it more convenient and adaptable in applications, Nelson [11]. Many researchers have investigated repairable systems using Copula techniques, but little has been provided on the k-out-of-nsystem. To name a few, Gahlot et al. [7] examined a system made up of three identical units using the k-out-of-n: G policy with copula repair methodology. Poonia and Sirohi [14] conducted a cost-benefit analysis of a heated standby system using the k-out-of-n: G under catastrophic failure. Yusuf et al. [27] examined a system with five clients and two servers as subsystems 1 and 2 under the k-out-of-n: G and discussed the consequences of copula repairs. Poonia et al. [15] used the copula repair approach to examine the performance of a warm standby k-out-of-n: G and 2-out-of-4: G system. Singh et al. [22] reported a reliability assessment of a multi-computer system with n clients and the k-out-of-n: G operation scheme with copula repair policy.To analyze the availability of the butteroil production system, Mehta et al [31] developed a mathematical model that took into account the reliability of each subsystem and the system as a whole. The model also considered the time required for repair and maintenance of the subsystems. According to the study, the development of a mathematical model for analyzing the availability of industrial systems such as the butteroil production system is important for improving the reliability and efficiency of such systems, which can lead to cost savings and increased productivity. Kumar and Ram [32] demonstrates the importance of reliability modeling in ensuring the efficient and effective operation of industrial plants such as sugar mills. By using Markov processes to develop reliability models, they were able to analyze the performance metrics of the sugar mill plant and optimize its maintenance schedule, thereby enhancing its reliability and profitability. Yusuf et al [33] highlights the importance of developing effective maintenance strategies for multi-station manufacturing systems to ensure optimal performance and profitability. Kumar and Malik [36] provides a comprehensive analysis of the reliability of a computer system with two cold standby units, taking into account both hardware and software failures, as well as the role of a maintenance server in ensuring system reliability

Many literatures on system performance evaluations employing the k-out-of-n: G scheme redundancy studied traditional measures of repairable systems. Such literatures present different types of system architectures employing the k-out-of-n: G scheme in form of standby, series or parallel configurations. Motivated by the work ofPoonia and Sirohi [14], Poonia et al.[21], Singh and Ram [19], Singh and Gahlot [20], Singh et al. [21] and Singh et al. [22], in which the system design consisting of only one subsystem with a k-out-of-n: G operational scheme. Little is known on performance evaluation of serial system consisting of two subsystems in which each subsystem possesses n units working under k-out-of-n: G scheme attended by a controller whose error can lead to system failure. In this study, the performance of the series-parallel system with two subsystems each using k-out-of-n is analyzed using Copula characteristics: Controller for the G-schema. The objectives are to determine the most efficient repair approach that will increase system effectiveness and the system's most crucial components.

This paper is structured as follows: Section 1 contains the introduction as well as a brief review of the literature. Notations, assumptions, and system description are found in Section 2, whereas model formulation and solutions are found in Section 3. Section 4 discusses system analysis for specific scenarios, while the results were discussed in Section 5 and Section 6 brought the work to a close with references.

2. Notations, Assumptions and Model Description

2.1. Notations

n: number of components in the system.

k: minimum number of components that must work for the k-out-of-n:G systemto work.

q:Time variable on the time axis.

s:Laplace transform variable for each statement in the mathematical equations.

v_1: rate of failure of unit in subsystem A

v_2:rate of failure of unit in subsystem B

v_3:

v_c: rate of failure of unit due to controller mistake

$r_1(x)/r_2(y)$: rate of repair by general of unit in subsystem A/subsystem B

$r_0(x)/r_0(y)/r_0(z)$: rate copula repair for completely failed states

$F_i(q)$: For i =0 to 15, the probability that the system is instate S_i at any given period of time.

$\overline{F}(s)$: Laplace transformation of state transition probability $F(q)$.

F_i (x, q): The probability that a system is in a situation $S_i$where for i=1..., the system is undergoing repair, and the amount of time that has passed since the repair began is given by (x, t) wherexstands for repair and q for time.

F_i (y, q): The probability that a system is in state S_i where i=1..., indicates that it is undergoing repair, and where the elapsed repair time is given by the expression (y, q), where y denotes the repair process and q the passage of time.

F_i (z, q): The probability that a system is in state S_i where i=1..., the system is undergoing repair, and where the elapsed repair time is given by the expression (y, q), where y stands for the repair's duration and q for the amount of time.

$E_p(t)$: Expected profit over the course of the time interval [0, t).

H_1, H_2: Revenue and service cost per unit time, respectively.

$r_0(x)$: According to the Gumbel-Hougaard family copula definition, joint probability is expressed as: $c_\theta(u_1(x), u_2(x)) = exp\left(x^\theta + \{log\,\phi\,(x)^\theta\}^{\frac{1}{\theta}}\right), 1 \leq \theta \leq \infty$. Where $\mu_1 = \phi(x)$ and$u_2 = e^x$.

2.2. Assumptions

1. All components or subsystems are initially considered to be operational.
2. The controller and two parts from subsystem A and B are required for system operation.
3. Performance of the system is satisfactory when any unit fails.
4. Any unit that malfunctions can be repaired both in operation and in the failed state.
5. It is assumed that all failure rates are constant and have an exponential distribution.

2.3. Model Description

The system under consideration is a serial system consisting of two subsystems. Each of the subsystem has n number of units working under the k-out-of-n: G scheme. The entire system operation is attended or under the control of human controller. Thus, the human controller is in charge of smooth operation of the system. When any of the unit failed in any of the subsystem, it is substituted by any of the standby among the remaining n-k units. The system failure occurred when all n units have failed in any of the subsystem or due to error of human controller. The description of each state is given immediately after figure 1.

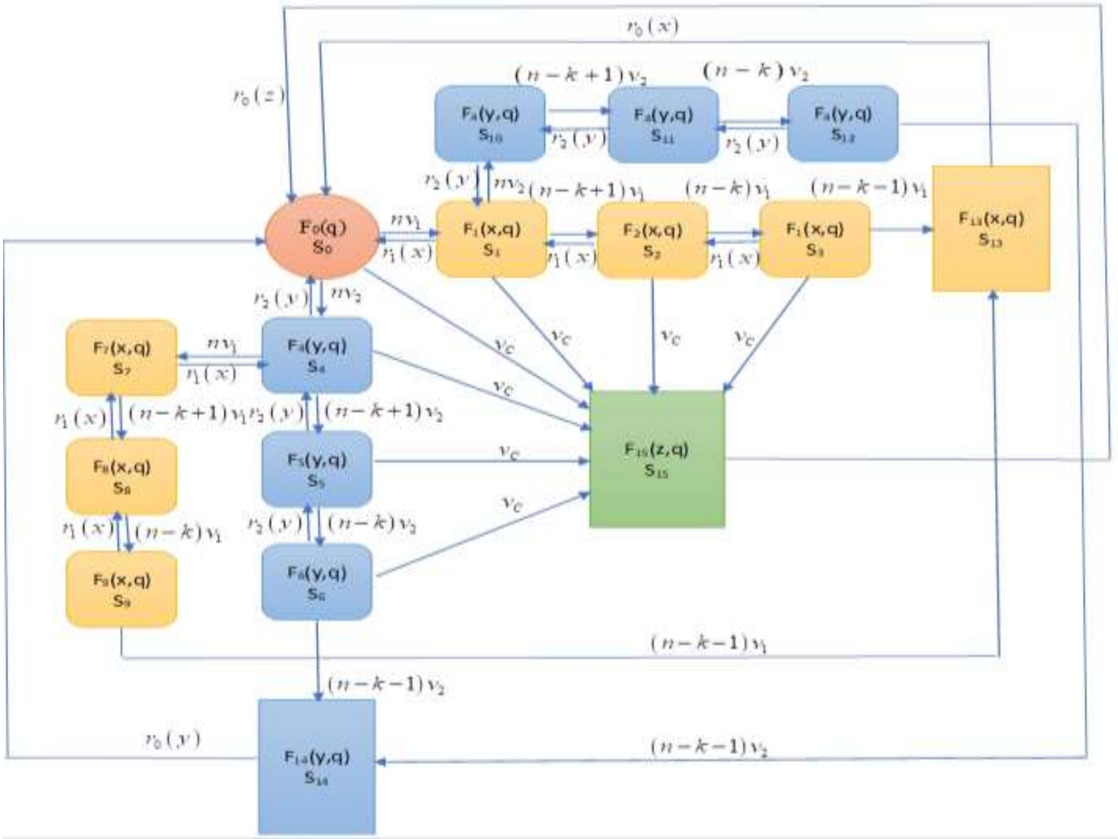

Figure 1. Transition diagram of the model

S_0: Initial or Perfect state of the system where the system does not experience any failure. The system is up and running.

S_1: First unit failure in subsystem A among n units and is under repair by general repair. The system is up and running.

S_2: Second unit failure in subsystem A among $(n-k+1)$ units and is under repair by general repair. The system is up and running.

S_3: Third unit failure in subsystem A among $(n-k)$ units and is under repair by general repair. The system is up and running.

S_4: First unit failure in subsystem B among n units and is under repair by general repair. The system is up and running.

S_5: Second unit failure in subsystem B among $(n-k+1)$ units and is under repair by general repair. The system is up and running.

S_6: Third unit failure in subsystem B among $(n-k)$ units and is under repair by general repair. The system is up and running.

S_7:Previously first unit has failed in subsystem B among n units and is under repair by general repair, followed by another unit failure in subsystem A among n units. The system is up and running.

S_8:Previously first unit has failed in subsystem B among n unit and is under repair by general repair, followed by another unit failure in subsystem A among $(n-k+1)$ units. The system is up and running.

S_9:Previously first unit has failed in subsystem B among n unit and is under repair by general repair, followed by another unit failure in subsystem A among $(n-k)$ units. The system is up and running.

S_{10}:Previously first unit has failed in subsystem A among n units and is under repair by general repair, followed by another unit failure in subsystem B among n units. The system is up and running.

S_{11}:Previously first unit has failed in subsystem A among n unit and is under repair by general repair, followed by another unit failure in subsystem B among $(n-k+1)$ units. The system is up and running.

S_{12}:Previously first unit has failed in subsystem A among n unit and is under repair by general repair, followed by another unit failure in subsystem B among $(n-k)$ units. The system is up and running.

S_{13}: System is down due to failure of $(n-k-1)$units in subsystem A.

S14: System is down due to failure of $(n-k-1)$ units in subsystem B.

S15: System is down due to human controller error.

3. Formulation of Model and Solution

3.1. Model Formulation

We can derive the following set of different differential equations regulating the current mathematical model using probability of considerations and continuity arguments. The following partial differential equations obtained via Figure 1 through approach of probability and solving alongside with their corresponding initial and boundary conditions associated with model's development are derived from Figure 1 above and solved using Laplace transformation to generate the state probabilities. (see Yusuf et al. (2022)).

$$\left(\frac{\delta}{\delta q}+nv_1+nv_2+v_c\right)F_0(q)=\int_0^\infty r_1(x)F_1(x,q)dx+\int_0^\infty r_2(x)F_4(y,q)dy+\int_0^\infty r_0(x)F_{13}(x,q)dx+\int_0^\infty r_0(y)F_{14}(y,q)dy+\int_0^\infty r_0(z)F_{15}(z,q)dz \tag{1}$$

$$\left(\frac{\delta}{\delta q}+\frac{\delta}{\delta x}+(n-k+1)v_1+nv_2+v_c+r_1(x)\right)F_1(x,q)=0 \tag{2}$$

$$\left(\frac{\delta}{\delta q}+\frac{\delta}{\delta x}+(n-k)v_1+v_c+r_1(x)\right)F_2(x,q)=0 \tag{3}$$

$$\left(\frac{\delta}{\delta q}+\frac{\delta}{\delta x}+(n-k-1)v_1+v_c+r_1(x)\right)F_3(x,q)=0 \tag{4}$$

$$\left(\frac{\delta}{\delta q}+\frac{\delta}{\delta y}+(n-k+1)v_2+nv_1+v_c+r_2(y)\right)F_4(y,q)=0 \tag{5}$$

$$\left(\frac{\delta}{\delta q}+\frac{\delta}{\delta y}+(n-k)v_2+v_c+r_2(y)\right)F_5(y,q)=0 \tag{6}$$

$$\left(\frac{\delta}{\delta q}+\frac{\delta}{\delta y}+(n-k-1)v_2+v_c+r_2(y)\right)F_6(y,q)=0 \tag{7}$$

$$\left(\frac{\delta}{\delta q}+\frac{\delta}{\delta x}+(n-k+1)v_1+r_1(x)\right)F_7(x,q)=0 \tag{8}$$

$$\left(\frac{\delta}{\delta q}+\frac{\delta}{\delta x}+(n-k)v_1+r_1(x)\right)F_8(x,q)=0 \tag{9}$$

$$\left(\frac{\delta}{\delta q}+\frac{\delta}{\delta x}+(n-k-1)v_1+r_1(x)\right)F_9(x,q)=0 \tag{10}$$

$$\left(\frac{\delta}{\delta q}+\frac{\delta}{\delta y}+(n-k+1)v_2+r_2(y)\right)F_{10}(y,q)=0 \tag{11}$$

$$\left(\frac{\delta}{\delta q}+\frac{\delta}{\delta y}+(n-k)v_2+r_2(y)\right)F_{11}(y,q)=0 \tag{12}$$

$$\left(\frac{\delta}{\delta q}+\frac{\delta}{\delta y}+(n-k-1)v_2+r_2(y)\right)F_{12}(y,q)=0 \tag{13}$$

$$\left(\frac{\delta}{\delta q}+\frac{\delta}{\delta x}+r_0(x)\right)F_{13}(x,q)=0 \tag{14}$$

$$\left(\frac{\delta}{\delta q}+\frac{\delta}{\delta y}+r_0(y)\right)F_{14}(y,q)=0 \tag{15}$$

$$\left(\frac{\delta}{\delta q}+\frac{\delta}{\delta z}+r_0(z)\right)F_{15}(z,q)=0 \quad (16)$$

3.2. Boundary conditions

$$F_1(0,q)=nv_1F_0(q) \quad (17)$$
$$F_2(0,q)=n(n-k+1)v_1^2F_0(q) \quad (18)$$
$$F_3(0,q)=n(n-k+1)(n-k)v_1^3F_0(q) \quad (19)$$
$$F_4(0,q)=nv_2F_0(q) \quad (20)$$
$$F_5(0,q)=n(n-k+1)v_2^2F_0(q) \quad (21)$$
$$F_6(0,q)=n(n-k+1)(n-k)v_2^3F_0(q) \quad (22)$$
$$F_7(0,q)=n^2v_1v_2F_0(q) \quad (23)$$
$$F_8(0,q)=n^2(n-k+1)v_1^2v_2F_0(q) \quad (24)$$
$$F_9(0,q)=n^2(n-k+1)(n-k)v_1^3v_2F_0(q) \quad (25)$$
$$F_{10}(0,q)=n^2v_1v_2F_0(q) \quad (26)$$
$$F_{11}(0,q)=n^2(n-k+1)v_1v_2^2F_0(q) \quad (27)$$
$$F_{12}(0,q)=n^2(n-k+1)(n-k)v_1v_2^3F_0(q) \quad (28)$$
$$F_{13}(0,q)=n(n-k+1)(n-k)(n-k-1)v_1^4(1+nv_2)F_0(q) \quad (29)$$
$$F_{14}(0,q)=n(n-k+1)(n-k)(n-k-1)v_2^4(1+nv_1)F_0(q) \quad (30)$$
$$F_{15}(0,q)=v_c\left(1+nv_1+nv_2+n(n-k+1)v_1^2+n(n-k+1)v_2^2+n(n-k+1)(n-k)v_1^3+n(n-k+1)(n-k)v_2^3\right)F_0(q) \quad (31)$$

3.3. Model Solution

With the help of initial condition $F(0)=1$and other state probabilities are zero at q= 0, we can obtain the Laplace transforms of equations (1)-(31) as:

$$(s+nv_1+nv_2+v_c)\overline{F}_0(q)=1+\int_0^\infty r_1(x)\overline{F}_1(x,s)dx+\int_0^\infty r_2(y)\overline{F}_4(y,s)dy+\int_0^\infty r_0(x)\overline{F}_{13}(x,s)dx+$$
$$\int_0^\infty r_0(y)\overline{F}_{14}(y,s)dy+\int_0^\infty r_0(z)\overline{F}_{15}(z,s)dz \quad (32)$$

$$\left(s+\frac{\delta}{\delta x}+(n-k+1)v_1+nv_2+v_c+r_1(x)\right)\overline{F}_1(x,s)=0 \quad (33)$$

$$\left(s+\frac{\delta}{\delta x}+(n-k)v_1+v_c+r_1(x)\right)\overline{F}_2(x,s)=0 \quad (34)$$

$$\left(s+\frac{\delta}{\delta x}+(n-k-1)v_1+v_c+r_1(x)\right)\overline{F}_3(x,s)=0 \quad (35)$$

$$\left(s+\frac{\delta}{\delta y}+(n-k+1)v_2+nv_1+v_c+r_2(y)\right)\overline{F}_4(y,s)=0 \quad (36)$$

$$\left(s+\frac{\delta}{\delta y}+(n-k)v_2+v_c+r_2(y)\right)\overline{F}_5(y,s)=0 \quad (37)$$

$$\left(s+\frac{\delta}{\delta y}+(n-k-1)v_2+v_c+r_2(y)\right)\overline{F}_6(y,s)=0 \tag{38}$$

$$\left(s+\frac{\delta}{\delta x}+(n-k+1)v_1+r_1(x)\right)\overline{F}_7(x,s)=0$$

(39)

$$\left(s+\frac{\delta}{\delta x}+(n-k)v_1+r_1(x)\right)\overline{F}_8(x,s)=0 \tag{40}$$

$$\left(s+\frac{\delta}{\delta x}+(n-k-1)v_1+r_1(x)\right)\overline{F}_9(x,s)=0 \tag{41}$$

$$\left(s+\frac{\delta}{\delta y}+(n-k+1)v_2+r_2(y)\right)\overline{F}_{10}(y,s)=0 \tag{42}$$

$$\left(s+\frac{\delta}{\delta y}+(n-k)v_2+r_2(y)\right)\overline{F}_{11}(y,s)=0 \tag{43}$$

$$\left(s+\frac{\delta}{\delta y}+(n-k-1)v_2+r_2(y)\right)\overline{F}_{12}(y,s)=0 \tag{44}$$

$$\left(s+\frac{\delta}{\delta x}+r_0(x)\right)\overline{F}_{13}(x,s)=0 \tag{45}$$

$$\left(s+\frac{\delta}{\delta y}+r_0(y)\right)\overline{F}_{13}(y,s)=0 \tag{46}$$

$$\left(s+\frac{\delta}{\delta z}+r_0(z)\right)\overline{F}_{15}(z,s)=0 \tag{47}$$

3.4. Laplace transformation of Boundary conditions

$$\overline{F}_1(0,q)=nv_1\overline{F}_0(s) \tag{48}$$

$$\overline{F}_2(0,q)=n(n-k+1)v_1^2\overline{F}_0(s) \tag{49}$$

$$\overline{F}_3(0,q)=n(n-k+1)(n-k)v_1^3\overline{F}_0(s) \tag{50}$$

$$\overline{F}_4(0,q)=nv_2\overline{F}_0(s) \tag{51}$$

$$\overline{F}_5(0,q)=n(n-k+1)v_2^2\overline{F}_0(s) \tag{52}$$

$$\overline{F}_6(0,q)=n(n-k+1)(n-k)v_2^3\overline{F}_0(s) \tag{53}$$

$$\overline{F}_7(0,q)=n^2v_1v_2\overline{F}_0(s) \tag{54}$$

$$\overline{F}_8(0,q)=n^2(n-k+1)v_1^2v_2\overline{F}_0(s) \tag{55}$$

$$\overline{F}_9(0,q)=n^2(n-k+1)(n-k)v_1^3v_2\overline{F}_0(s) \tag{56}$$

$$\overline{F}_{10}(0,q)=n^2v_1v_2\overline{F}_0(s) \tag{57}$$

$$\overline{F}_{11}(0,q)=n^2(n-k+1)v_1v_2^2\overline{F}_0(s) \tag{58}$$

$$\overline{F}_{12}(0,q)=n^2(n-k+1)(n-k)v_1v_2^3\overline{F}_0(s) \tag{59}$$

$$\overline{F}_{13}(0,q)=n(n-k+1)(n-k)(n-k-1)v_1^4(1+nv_2)\overline{F}_0(s) \tag{60}$$

$$\overline{F}_{14}(0,q)=n(n-k+1)(n-k)(n-k-1)v_2^4(1+nv_1)\overline{F}_0(s) \tag{61}$$

$$\overline{F}_{15}(0,q)=v_c\left(1+nv_1+nv_2+n(n-k+1)v_1^2+n(n-k+1)v_2^2+n(n-k+1)(n-k)v_1^3+n(n-k+1)(n-k)v_2^3\right)\overline{F}_0(s) \quad (62)$$

Solving equations (32)-(47) simultaneously by substituting equations (48)-(62), we obtain:

$$\overline{F}_0(0)=\frac{1}{G(s)}$$

$$\overline{F}_1(s)=nv_1\left(\frac{1-\overline{s}_r\left(s+(n-k+1)v_1+nv_2+v_c\right)}{s+(n-k+1)v_1+nv_2+v_c}\right)\overline{F}_0(s) \quad (63)$$

$$\overline{F}_2(s)=n(n-k+1)v_1^2\left(\frac{1-\overline{s}_r\left(s+(n-k)v_1+v_c\right)}{s+(n-k)v_1+v_c}\right)\overline{F}_0(s) \quad (64)$$

$$\overline{F}_3(s)=n(n-k+1)(n-k)v_1^3\left(\frac{1-\overline{s}_r\left(s+(n-k-1)v_1+v_c\right)}{s+(n-k-1)v_1+v_c}\right)\overline{F}_0(s) \quad (65)$$

$$\overline{F}_4(s)=nv_1\left(\frac{1-\overline{s}_r\left(s+(n-k+1)v_2+nv_1+v_c\right)}{s+(n-k+1)v_2+nv_1+v_c}\right)\overline{F}_0(s) \quad (66)$$

$$\overline{F}_5(s)=n(n-k+1)v_2^2\left(\frac{1-\overline{s}_r\left(s+(n-k)v_2+v_c\right)}{s+(n-k)v_2+v_c}\right)\overline{F}_0(s) \quad (67)$$

$$\overline{F}_6(s)=n(n-k+1)(n-k)v_2^3\left(\frac{1-\overline{s}_r\left(s+(n-k)v_2+v_c\right)}{s+(n-k)v_2+v_c}\right)\overline{F}_0(s) \quad (68)$$

$$\overline{F}_7(s)=n^2v_1v_2\left(\frac{1-\overline{s}_r\left(s+(n-k+1)v_1\right)}{s+(n-k+1)v_1}\right)\overline{F}_0(s) \quad (69)$$

$$\overline{F}_8(s)=n^2(n-k+1)v_1^2v_2\left(\frac{1-\overline{s}_r\left(s+(n-k)v_1\right)}{s+(n-k)v_1}\right)\overline{F}_0(s) \quad (70)$$

$$\overline{F}_9(s)=n^2(n-k+1)v_1^3v_2\left(\frac{1-\overline{s}_r\left(s+(n-k-1)v_1\right)}{s+(n-k-1)v_1}\right)\overline{F}_0(s) \quad (71)$$

$$\overline{F}_{10}(s)=n^2v_1v_2\left(\frac{1-\overline{s}_r\left(s+(n-k+1)v_2\right)}{s+(n-k+1)v_2}\right)\overline{F}_0(s) \quad (72)$$

$$\overline{F}_{11}(s)=n^2(n-k+1)v_1v_2^2\left(\frac{1-\overline{s}_r\left(s+(n-k)v_2\right)}{s+(n-k)v_2}\right)\overline{F}_0(s) \quad (73)$$

$$\overline{F}_{12}(s)=n^2(n-k+1)(n-k)v_1v_2^3\left(\frac{1-\overline{s}_r\left(s+(n-k-1)v_2\right)}{s+(n-k-1)v_2}\right)\overline{F}_0(s) \quad (74)$$

$$\overline{F}_{13}(s)=n^2(n-k+1)(n-k)(n-k-1)v_1^4(1+nv_2)\left(\frac{1-\overline{s}_{r_0}(s)}{s}\right)\overline{F}_0(s) \quad (75)$$

$$\overline{F}_{14}(s)=n^2(n-k+1)(n-k)(n-k-1)v_2^4(1+nv_1)\left(\frac{1-\overline{s}_{r_0}(s)}{s}\right)\overline{F}_0(s) \quad (76)$$

$$\overline{F}_{15}(s)=v_c\left\{\begin{matrix}1+nv_1+nv_2+n(n-k+1)v_1^2+n(n-k+1)v_2^2\\+n(n-k+1)(n-k)v_2^3+n(n-k+1)(n-k)v_1^3\end{matrix}\right\}\left(\frac{1-\overline{s}_{r_0}(s)}{s}\right)\overline{F}_0(s) \quad (77)$$

Where;

$$G(s)=\left\{\begin{array}{l}(s+nv_1+nv_2+v_c)-nv_1\bar{s}_r\left(s+(n-k+1)v_1+nv_2+v_c\right)\\ -nv_2\bar{s}_r\left(s+(n-k+1)v_2+nv_1+v_c\right)-\\ v_c\left\{\begin{array}{l}1+nv_1+nv_2+n(n-k+1)v_1^2+n(n-k+1)v_2^2\\ +n(n-k+1)(n-k)v_2^3+n(n-k+1)(n-k)v_1^3\end{array}\right\}\bar{s}_{r_0}(s)-\\ n(n-k+1)(n-k)(n-k-1)v_1^4(1+nv_2)\bar{s}_{r_0}(s)-\\ n(n-k+1)(n-k)(n-k-1)v_2^4(1+nv_1)\bar{s}_{r_0}\end{array}\right\}$$

The following are the Laplace transformations of the probabilities that the system is up and failing at any given time:

$$\bar{F}_{up}(s)=\left\{\begin{array}{l}\bar{F}_0(s)+\bar{F}_1(s)+\bar{F}_2(s)+\bar{F}_3(s)+\bar{F}_4(s)+\bar{F}_5(s)+\\ \bar{F}_6(s)+\bar{F}_7(s)+\bar{F}_8(s)+\bar{F}_9(s)+\bar{F}_{11}(s)+\bar{F}_{12}(s)\end{array}\right\} \quad (78)$$

$$\bar{F}_{up}(s)=\frac{1}{G(s)}\left\{\begin{array}{l}1+nv_1\left(\frac{1-\bar{s}_r(s+y_1v_1+nv_2+v_c)}{s+y_1v_1+nv_2+v_c}\right)+nv_2\left(\frac{1-\bar{s}_r(s+y_1v_2+nv_1+v_c)}{s+y_1v_2+nv_1+v_c}\right)+\\ n(n-k+1)(n-k)v_1^3\left(\frac{1-\bar{s}_r(s+y_3v_1+v_c)}{s+y_3v_1+v_c}\right)+n(n-k+1)v_1^2\left(\frac{1-\bar{s}_r(s+y_2v_1+v_c)}{(s+y_2v_1+v_c)}\right)+\\ n(n-k+1)v_2^2\left(\frac{1-\bar{s}_r(s+y_2v_2+v_c)}{(s+y_2v_2+v_c)}\right)+n(n-k+1)(n-k)v_2^3\left(\frac{1-\bar{s}_r(s+y_3v_2+v_c)}{(s+y_3v_2+v_c)}\right)+\\ n^2v_1v_2\left(\frac{1-\bar{s}_r(s+y_1v_1)}{(s+y_1v_1)}\right)+n^2(n-k+1)v_1^2v_2\left(\frac{1-\bar{s}_r(s+y_2v_1)}{(s+y_2v_1)}\right)+\\ n^2y_1y_2v_1^3v_2\left(\frac{1-\bar{s}_r(s+y_1v_1)}{(s+y_1v_1)}\right)+n^2v_1v_2\left(\frac{1-\bar{s}_r(s+y_1v_2)}{(s+y_1v_2)}\right)+\\ n^2y_1v_1v_2^2\left(\frac{1-\bar{s}_r(s+y_2v_2)}{(s+y_2v_2)}\right)+n^2y_1y_2v_1v_2^3\left(\frac{1-\bar{s}_r(s+y_3v_2)}{(s+y_3v_2)}\right)\end{array}\right\} \quad (79)$$

4. Model analysis for specific cases

4.1. System Availability Analysis

4.1.1. Availability Analysis for Copula Repair

To obtain the availability of the system, we set $\bar{S}_r(s)=\frac{r}{s+r}$, $\frac{1-\bar{S}_r(s)}{s}=\frac{1}{s+r}$, $\bar{S}_{r_0}(s)=\bar{S}_{exp[x^\theta+\{log\emptyset(x)\}^\theta]^{1/\theta}}(S)=\frac{exp[x^\theta+\{log\emptyset(x)\}^\theta]^{1/\theta}}{s+exp[x^\theta+\{log\emptyset(x)\}^\theta]^{1/\theta}}$, $v_1=0.001, v_2=0.002,\ v_3=0.003, v_c=0.004$ and all repairs to 1 i.e. $r_1(x)=r_2(y)=r_0(x)=r_0(y)=r_0(z)=1$ in (79) andinverse Laplace transform, the expression for system availability can be derived as:

$k=5$ and $n=10$.

$$F_{up}(q)=-8.78297971410e^{-1.01q}-1.32144550810e^{-1.004q}-0.00002277718358e^{-1.009q}+0.001539631406e^{-2.722494532q}-0.001228275893e^{-1.056605286q}-0.000005387798694e^{-1.028543017q}+0.9999432223e^{-0.0006571647893q}-1.1782236310e^{-1.008q}-5.15039221310e^{-1.005q}-0.00008350001402e^{-1.06q}-0.00006817816256e^{-1.012q}-0.00007321001780e^{-1.014q} \quad (80)$$

$k = 7$ and $n = 10$.

$$F_{up}(t) = -0.000049925443355e^{-1.01q} - 0.00001434170084e^{-1.007q} - 0.00007986225402e^{-1.004q} -0.00006907928738e^{-1.008q} - 5.08659508710e^{-1.002q} - 6.41052567710e^{-1.006q} + 0.001539584104e^{-2.722494048q} - 0.001077075793e^{-1.053523254q} - 0.000009301899033e^{-1.025717638q} +0.9997609780e^{-0.0005650608747q} - 3.29467312010e^{-1.003q} \quad (81)$$

$k = 9$ and $n = 10$

$$F_{up}(q) = 0.001539541320e^{-2.722493579q} - 0.0009272087420e^{-1.050509078q} +0.9995776366e^{-0.0004723206318q} - 0.00007000156272e^{-1.004q} - 0.00007602626134e^{-1.002q} -0.000006684068885e^{-1.005q} - 0.00002543552082e^{-1.006q} - 1.556863191910e^{-1.001q} -8.10292383010e^{-1.00q} - 0.00001166373589e^{-1.022825022q} \quad (82)$$

Using varying numbers for time $q = 0,5,10,20,30,40,50,60,70,80, and\ 90$ and $k = 5,7,9, n = 10$. Table 1 and figure 2 show the system's availability when Copula distribution is employed.

Table 1. System Availability against Time for Copula Repair ($k = 5,7,9, n = 10$)

q	Availability $k = 5\ and\ n = 10$	Availability $k = 7\ and\ n = 10$	Availability $k = 9\ and\ n = 10$
0	1.00000	1.00000	1.00000
5	0.99666	0.99693	0.99721
10	0.99339	0.99413	0.99487
20	0.98689	0.98853	0.99018
30	0.98042	0.98296	0.98551
40	0.97400	0.97742	0.98087
50	0.96762	0.97191	0.97625
60	0.96128	0.96643	0.97165
70	0.95499	0.96099	0.96707
80	0.94873	0.95557	0.96251
90	0.94252	0.95019	0.95798

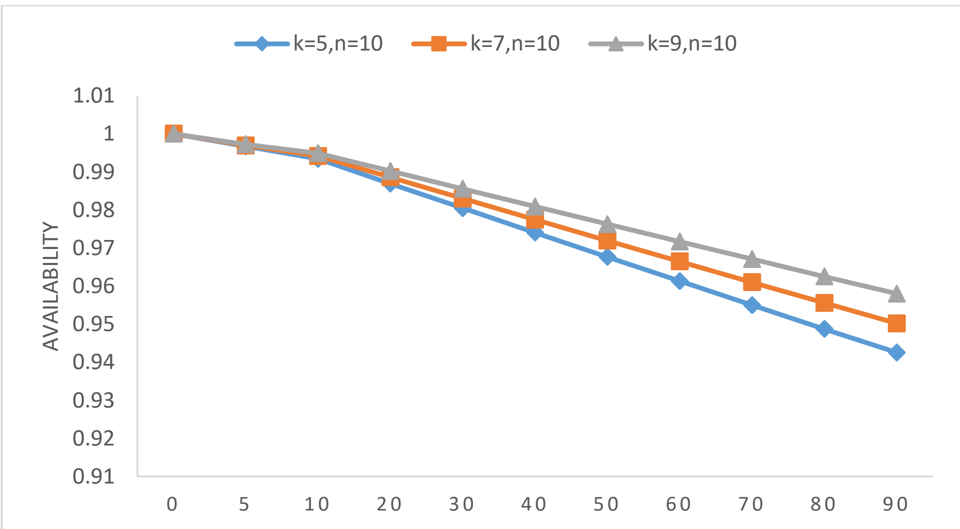

Figure 2. Availability against time for copula repair for different k

$k = 5$ and $n = 15$

$$F_{up}(t) = -0.000002093934792e^{-1.010q} - 0.0001867345851e^{-1.011q} - 1.06643347510e^{-1.009q} - 0.0001907126575e^{-1.024q} - 0.0001442619674e^{-1.022q} + 0.001575566382e^{-2.722596607q} - 0.002776372903e^{-1.085542048q} - 0.000006387507382e^{-1.043585278q} + 1.001799340e^{-0.001576067692q} - 6.57148662910e^{-1.013q} - 0.00006428601144e^{-1.014q} - 0.000003293119875e^{-1.020q} \quad (83)$$

$k = 5$ and $n = 20$

$$F_{up}(q) = -0.5383052153e^{-1.015q} - 0.0003314923886e^{-1.016q} - 0.000008055463350e^{-1.03q} - 0.000001928010504e^{-1.018q} + 0.001612179406e^{-2.722702616q} - 0.004859226483e^{-1.114143520q} - 0.000006999733126e^{-1.058606026q} + 1.004336720e^{-0.002847837185q} - 0.0001264481828e^{-1.019q} - 0.0002498361272e^{-1.032q} - 4.09567049510e^{-1.014q} - 0.0003591200178e^{-1.034q} \quad (84)$$

$k = 5$ and $n = 25$

$$F_{up}(q) = -0.00001593237792e^{-1.04q} - 0.0002095015761e^{-1.024q} - 0.00004245248090e^{-1.023q} - 0.0005182694978e^{-1.021q} - 0.000001114711909e^{-1.019q} + 0.001649707967e^{-2.722813489q} - 0.007423475294e^{-1.14241953q} - 0.000007429290047e^{-1.073618347q} + 1.007506215e^{-0.004446211615q} - 0.0003864751224e^{-1.042q} - 0.0005784633122e^{-1.044q} - 0.00001101625161e^{-1.02q} \quad (85)$$

Using varying numbers for time $t = 0,5,10,20,30,40,50,60,70,80, and\ 90$ and $k = 5, n = 15,20,25$. Table 2 and figure 3 show the system's availability when Copula distribution is used.

Table 2. System Availability against Time for Copula Repair ($k = 5, n = 15,20,25$)

Time	Availability $k = 5\ and\ n = 15$	Availability $k = 5\ and\ n = 20$	Availability $k = 5\ and\ n = 25$
0	1.00000	1.00000	1.00000
5	0.99392	0.99011	0.98532
10	0.98613	0.97614	0.96369
20	0.97071	0.94873	0.92178
30	0.95553	0.92209	0.88170
40	0.94059	0.89620	0.84335
50	0.92588	0.87104	0.80668
60	0.91141	0.84659	0.77160
70	0.89715	0.82282	0.73804
80	0.88313	0.79972	0.70594
90	0.86932	0.77726	0.67544

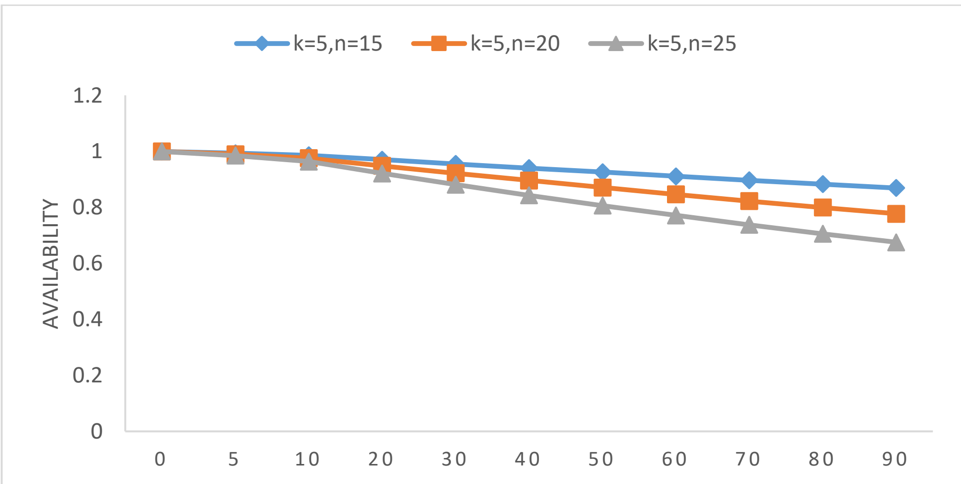

Figure 3. System availability against time for copula repair for different n

Table 3. Effect of Failure rates on System Availability against Time for Copula Repair ($k = 5$ and $n = 10$)

Time	Availability $k = 5$ and $n = 10$ $v_1 = 0.001, v_2 = 0.002, v_c = 0.004$	Availability $k = 5$ and $n = 10$ $v_1 = 0, v_2 = 0.002,$ $v_c = 0.004$	Availability $k = 5$ and $n = 10$ $v_1 = 0, v_2 = 0, v_c = 0.004$
0	1.00000	1.00000	1.00000
5	0.99666	0.99792	0.99853
10	0.99339	0.99678	0.99853
20	0.98689	0.99449	0.99853
30	0.98042	0.99222	0.99853
40	0.97400	0.98995	0.99853
50	0.96762	0.98768	0.99853
60	0.96128	0.98542	0.99853
70	0.95499	0.98316	0.99853
80	0.94873	0.98091	0.99853
90	0.94252	0.97867	0.99853

Table 4. Effect of Failure rates on System Availability against Time for Copula Repair ($k = 7$ and $n = 10$)

Time	Availability $k = 7$ and $n = 10$ $v_1 = 0.001, v_2 = 0.002, v_3 = 0.003, v_c = 0.004$	Availability $k = 7$ and $n = 10$ $v_1 = 0, v_2 = 0.002, v_3 = 0.003, v_c = 0.004$	Availability $k = 7$ and $n = 10$ $v_1 = 0, v_2 = 0, v_3 = 0.003, v_c = 0.004$	Availability $k = 7$ and $n = 10$ $v_1 = 0, v_2 = 0, v_3 = 0, v_c = 0.004$
0	1.00000	1.00000	1.00000	1.00000
5	0.99693	0.99814	0.99853	0.99853
10	0.99413	0.99738	0.99853	0.99853
20	0.98853	0.99585	0.99853	0.99853
30	0.98296	0.99433	0.99853	0.99853
40	0.97742	0.99281	0.99853	0.99853
50	0.97191	0.99129	0.99853	0.99853
60	0.96643	0.98977	0.99853	0.99853
70	0.96099	0.98825	0.99853	0.99853
80	0.95557	0.98674	0.99853	0.99853
90	0.95019	0.98523	0.99853	0.99853

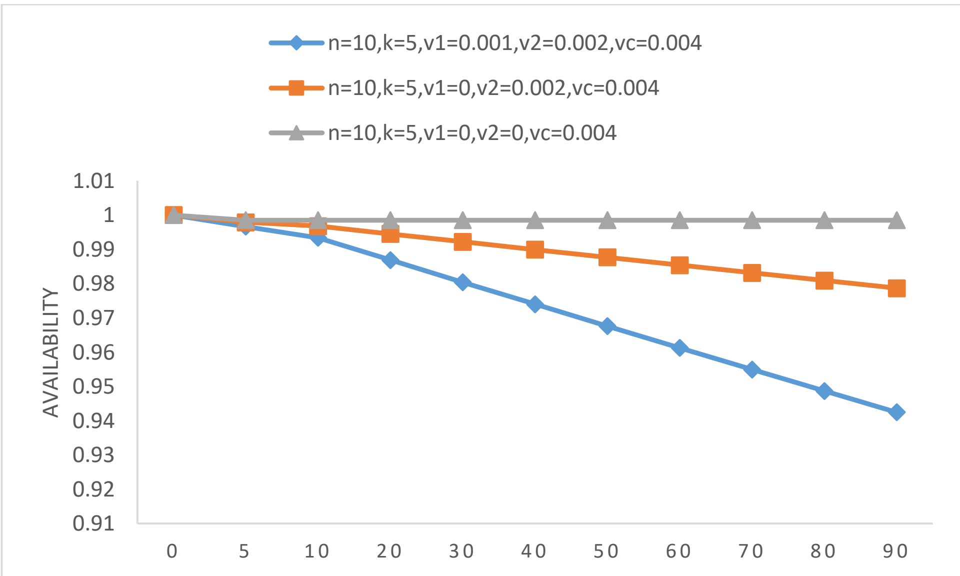

Figure 4. Effect of failure rates on availability against time for copula repair

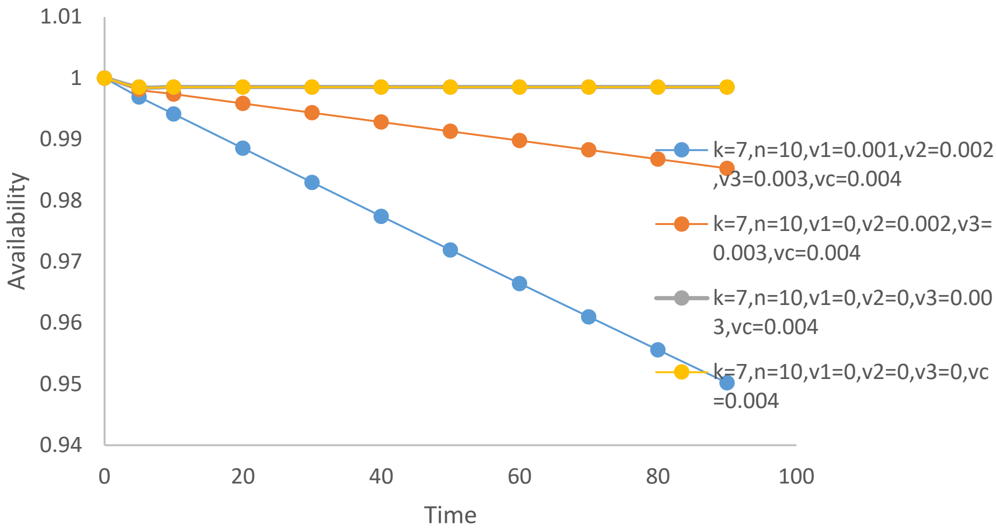

Figure 5. Effect of Failure rates on availability against time for copula repair

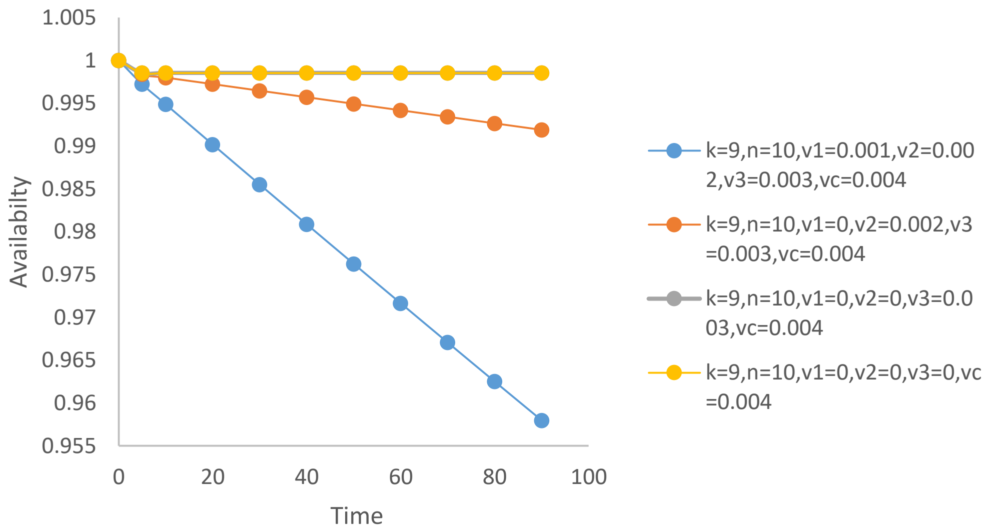

Figure 6. Effect of failure rates on availability against time for copula repair

Table 5. Effect of Failure rates on System Availability against Time for Copula Repair ($k = 9$ and $n = 10$)

Time	Availability $k = 9$ and $n = 10$ $v_1 = 0.001, v_2 = 0.002, v_3 = 0.003, v_c = 0.004$	Availability $k = 9$ and $n = 10$ $v_1 = 0, v_2 = 0.002, v_3 = 0.003, v_c = 0.004$	Availability $k = 9$ and $n = 10$ $v_1 = 0, v_2 = 0, v_3 = 0.003, v_c = 0.004$	Availability $k = 9$ and $n = 10$ $v_1 = 0, v_2 = 0, v_3 = 0, v_c = 0.004$
0	1.00000	1.00000	1.00000	1.00000
5	0.99721	0.99838	0.99853	0.99853
10	0.99487	0.99799	0.99853	0.99853
20	0.99018	0.99723	0.99853	0.99853
30	0.98551	0.99646	0.99853	0.99853
40	0.98087	0.99570	0.99853	0.99853
50	0.97625	0.99493	0.99853	0.99853
60	0.97165	0.99417	0.99853	0.99853
70	0.96707	0.99341	0.99853	0.99853
80	0.96251	0.99264	0.99853	0.99853
90	0.95798	0.99188	0.99853	0.99853

Table 6. Effect of Failure rates on System Availability against Time for Copula Repair ($k = 5$ and $n = 15$)

Time	Availability $k = 5$ and $n = 15$ $v_1 = 0.001, v_2 = 0.002, v_3 = 0.003, v_c = 0.004$	Availability $k = 5$ and $n = 15$ $v_1 = 0, v_2 = 0.002, v_3 = 0.003, v_c = 0.004$	Availability $k = 5$ and $n = 15$ $v_1 = 0, v_2 = 0, v_3 = 0.003, v_c = 0.004$	Availability $k = 5$ and $n = 15$ $v_1 = 0, v_2 = 0, v_3 = 0, v_c = 0.004$
0	1.00000	1.00000	1.00000	1.00000
5	0.99392	0.99852	0.99853	0.99853
10	0.98613	0.99849	0.99853	0.99853
20	0.97071	0.99843	0.99853	0.99853
30	0.95553	0.99836	0.99853	0.99853
40	0.94059	0.99830	0.99853	0.99853
50	0.92588	0.99823	0.99853	0.99853
60	0.91141	0.99817	0.99853	0.99853
70	0.89715	0.99811	0.99853	0.99853
80	0.88313	0.99804	0.99853	0.99853
90	0.86932	0.99798	0.99853	0.99853

Table 7. Effect of Failure rates on System Availability against Time for Copula Repair ($k = 5$ and $n = 20$)

Time	Availability $k = 5$ and $n = 20$ $v_1 = 0.001, v_2 = 0.002, v_3 = 0.003, v_c = 0.004$	Availability $k = 5$ and $n = 20$ $v_1 = 0, v_2 = 0.002, v_3 = 0.003, v_c = 0.004$	Availability $k = 5$ and $n = 20$ $v_1 = 0, v_2 = 0, v_3 = 0.003, v_c = 0.004$	Availability $k = 5$ and $n = 20$ $v_1 = 0, v_2 = 0, v_3 = 0, v_c = 0.004$
0	1.00000	1.00000	1.00000	1.00000
5	0.99011	0.99851	0.99853	0.99853
10	0.97614	0.99845	0.99853	0.99853
20	0.94873	0.99832	0.99853	0.99853
30	0.92209	0.99820	0.99853	0.99853
40	0.89620	0.99807	0.99853	0.99853
50	0.87104	0.99795	0.99853	0.99853
60	0.84659	0.99782	0.99853	0.99853
70	0.82282	0.99770	0.99853	0.99853
80	0.79972	0.99758	0.99853	0.99853
90	0.77726	0.99745	0.99853	0.99853

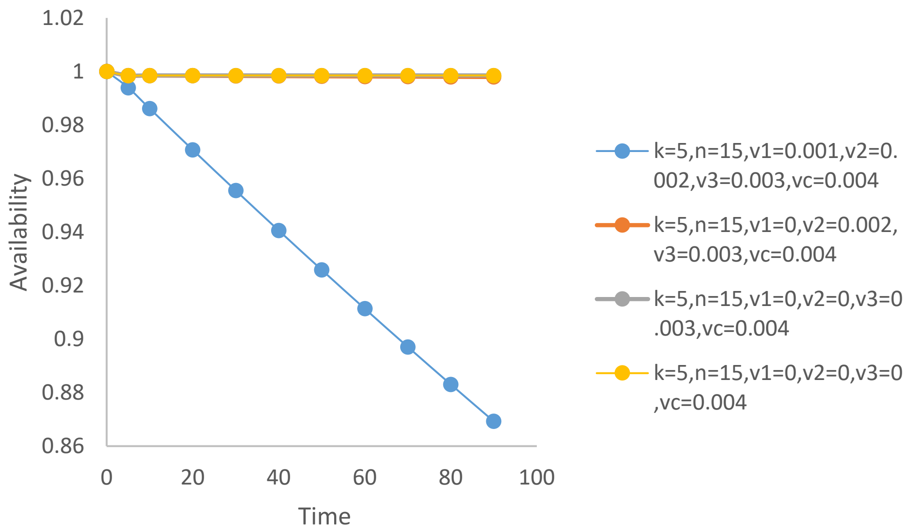

Figure 7. Effect of failure rates on availability against time for copula repair

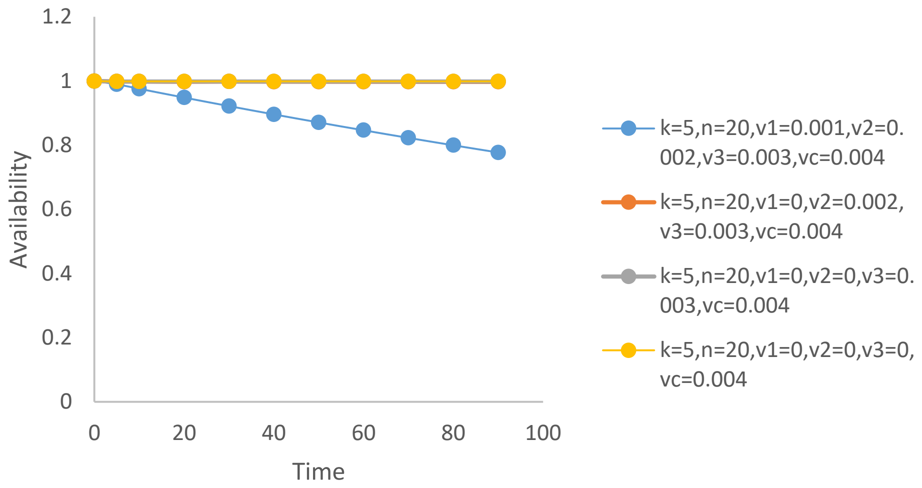

Figure 8. Effect of failure rates on availability against time for copula repair

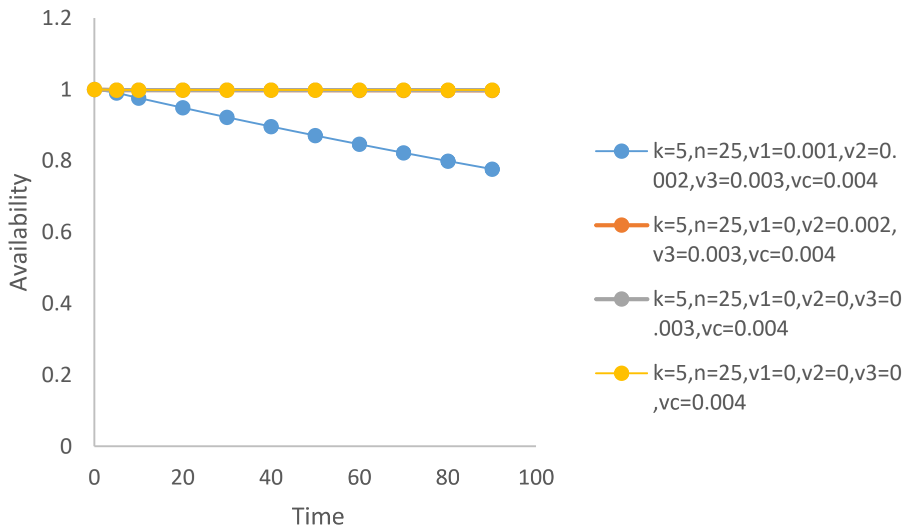

Figure 9. Effect of failure rates on availability against time for copula repair

Table 8. Effect of Failure rates on System Availability against Time for Copula Repair ($k = 5$ and $n = 25$)

Time	Availability $k = 5$ and $n = 25$ $v_1 = 0.001, v_2 = 0.002, v_3 = 0.003, v_c = 0.004$	Availability $k = 5$ and $n = 25$ $v_1 = 0, v_2 = 0.002, v_3 = 0.003, v_c = 0.004$	Availability $k = 5$ and $n = 25$ $v_1 = 0, v_2 = 0, v_3 = 0.003, v_c = 0.004$	Availability $k = 5$ and $n = 25$ $v_1 = 0, v_2 = 0, v_3 = 0, v_c = 0.004$
0	1.00000	1.00000	1.00000	1.00000
5	0.99011	0.99845	0.99853	0.99853
10	0.97614	0.99832	0.99853	0.99853
20	0.94873	0.99820	0.99853	0.99853
30	0.92209	0.99807	0.99853	0.99853
40	0.89620	0.99795	0.99853	0.99853
50	0.87104	0.99782	0.99853	0.99853
60	0.84659	0.99770	0.99853	0.99853
70	0.82282	0.99758	0.99853	0.99853
80	0.79972	0.99745	0.99853	0.99853
90	0.77726	0.99851	0.99853	0.99853

4.1.2. Availability Analysis for General Repair

After setting $\bar{S}_r(s) = \frac{r}{s+r}$in equation (79) and differentiating the parameters by giving them different values such as$v_1 = 0.001, v_2 = 0.002, v_3 = 0.003, v_c = 0.004$, and$r = 1$, one can obtain availability equation as follows by taking inverse the Laplace transform.

$n = 5$ and $k = 10$

$$F_{up}(t) == -0.000001037220279e^{-1.01q} - 0.00002765300049e^{-1.009q} - 2.43230403410e^{-1.004q} - 1.48184543810e^{-1.008q} - 8.00461776210e^{-1.005q} - 0.0001175229847e^{-1.006q} - 0.00008061013830e^{-1.014q} + 0.0009298612384e^{-1.058869190q} + 0.00001380183429e^{-1.028562257q} + 0.001945451845e^{-1.001913048q} + 0.9974159720e^{-0.0006555053320q} - 0.00007729052319e^{-1.012q} \quad (86)$$

$n = 7$ and $k = 10$

$$F_{up}(q) = -0.00005742709299e^{-1.01q} - 0.00001823289607e^{-1.007q} - 0.0001362791742e^{-1.004q} - 0.00008417277226e^{-1.008q} - 4.10661929310e^{-1.002q} - 8.64432790910e^{-1.006q} + 0.001210277173e^{-1.055906478q} + 0.00003952800804e^{-1.025766750q} + 0.001812458683e^{-1.00763138q} + 0.9972355171e^{-0.0005636345698q} - 0.763337827210e^{-1.003q} \quad (87)$$

$n = 9$ and $k = 10$

$$F_{up}(q) = -0.0001098070027e^{-1.004q} + 0.001495089445e^{-1.053016691q} + 0.001864779130e^{-1.001587344q} + 0.9970539656e^{-0.0004711288983q} - 0.0003495418229e^{-1.002q} - 0.000009252526030e^{-1.005q} - 0.00003262731610e^{-1.006q} + 2.53657401810e^{-1.001q} + 0.00008714114800e^{-1.022924836q} \quad (88)$$

Using varying numbers for time $t = 0,5,10,20,30,40,50,60,70,80,$ *and* 90 and $k = 5,7,9, n = 10$. Table 9 and figure 10 show the system's availability when general distribution is used.

Table 9. System Availability against Time for General Repair ($k = 5,7,9, n = 10$)

Time	Availability $k = 5$ *and* $n = 10$	Availability $k = 7$ *and* $n = 10$	Availability $k = 9$ *and* $n = 10$
0	1.00000	1.00000	1.00000
5	0.99417	0.99445	0.99473
10	0.99090	0.99163	0.99237
20	0.98443	0.98606	0.98770
30	0.97799	0.98051	0.98306
40	0.97160	0.97500	0.97844
50	0.96526	0.96952	0.97384
60	0.95895	0.96407	0.96926
70	0.95268	0.95866	0.96471
80	0.94646	0.95327	0.96017
90	0.94028	0.94791	0.95566

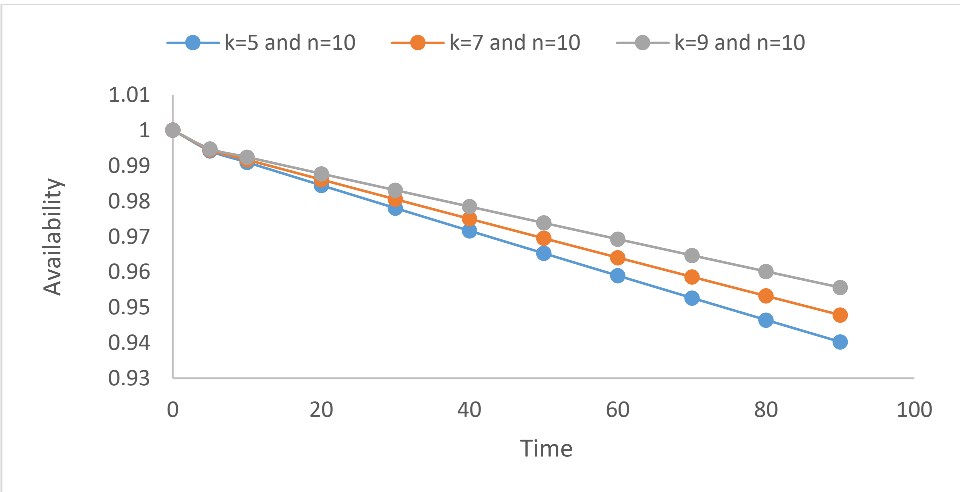

Figure 10. availability against time for general repair

$k = 5$ and $n = 15$

$$F_{up}(q) = -0.000002552925790e^{-1.01q} - 0.0002224586594e^{-1.011q} - 1.33832943210e^{-1.009q} - 0.0002012394969e^{-1.024q} - 0.0001536850100e^{-1.022q} - 0.0007210197116e^{-1.008784616q} + 0.000002177059185e^{-1.043593831q} + 0.002121062980e^{-1.002019480q} + 0.9992551252e^{-0.1572073393q} - 7.56061563210e^{-1.013q} - 0.00007297164130e^{-1.014q} - 0.000003547890347e^{-1.02q} \quad (89)$$

$k = 5$ and $n = 20$

$$F_{up}(q) = -0.000006130645274e^{-1.015q} - 0.002867821534e - 1.116451438q + 0.002264640045e^{-1.002097052q} + 1.0017688e^{-0.002840583018q} - 0.0003736463943e^{-1.016q} - 0.000002136336991e^{-1.032q} - 4.72033582610e^{-1.014q} - 0.0003729083184e^{-1.034q} - 0.000002083973668e^{-1.058610927q} - 0.000008450136411e^{1.03q} \quad (90)$$

$k = 5$ and $n = 25$

$$F_{up}(q) = -0.00001650772111e^{-1.04q} - 0.0002263652187e^{-1.024q} - 0.000004607576931e^{-1.023q} - 0.0005683065141e^{-1.021q} - 0.000001237684747e^{-1.019q} - 0.0003991632713e^{1.042q} - 0.005478581146e^{-1.144778627q} - 0.000004195938325e^{-1.07361567q} + 0.002399040085e^{-1.002164994q} + 1.0049077793^{-0.004434811174q} - 0.0005957024587e^{1.044q} - 0.00001215154210e^{-1.02q} \quad (91)$$

Using varying numbers for time $t = 0,5,10,20,30,40,50,60,70,80, and$ 90 and $k = 5, n = 15,20,25$. Table 10 and figure 11 show the system's availability when General distribution is used.

Table 10. System Availability against Time for General Repair ($k = 5, n = 15,20,25$)

Time	Availability $k = 5$ *and* $n = 15$	Availability $k = 5$ *and* $n = 20$	Availability $k = 5$ *and* $n = 25$
0	1.00000	1.00000	1.00000
5	0.99144	0.98764	0.98286
10	0.98367	0.97371	0.96132
20	0.96833	0.94644	0.91961
30	0.95322	0.91994	0.87972
40	0.93835	0.89417	0.84156
50	0.92372	0.86913	0.80505
60	0.90931	0.84479	0.77013
70	0.89513	0.82113	0.73672
80	0.88116	0.79813	0.70477
90	0.86742	0.77578	0.67419

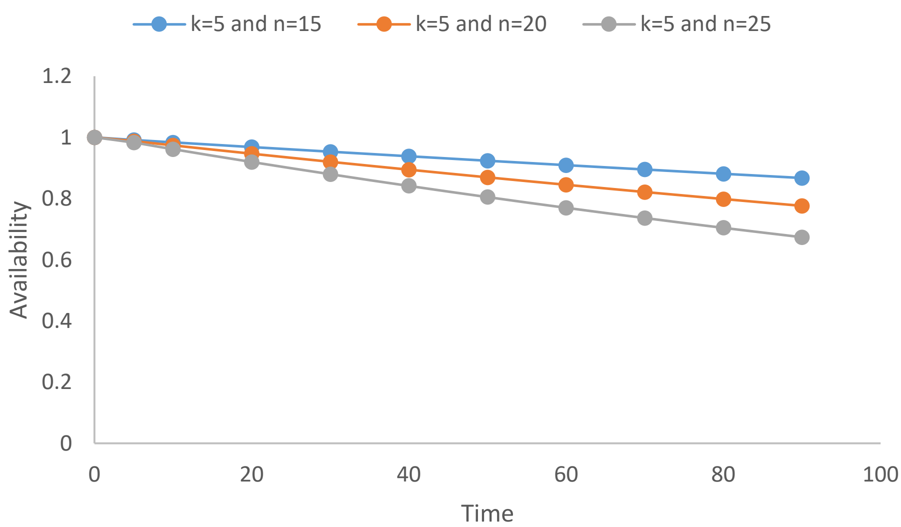

Figure 11. availability against time for general repair

Table 11. Effect of Failure rates on System Availability against Time for General Repair ($k = 5$ *and* $n = 10$)

Time	Availability $k = 5$ *and* $n = 10$ $v_1 = 0.001, v_2 = 0.002, v_3 = 0.003, v_c = 0.004$	Availability $k = 5$ *and* $n = 10$ $v_1 = 0, v_2 = 0.002, v_3 = 0.003, v_c = 0.004$	Availability $k = 5$ *and* $n = 10$ $v_1 = 0, v_2 = 0, v_3 = 0.003, v_c = 0.004$	Availability $k = 5$ *and* $n = 10$ $v_1 = 0, v_2 = 0, v_3 = 0, v_c = 0.004$
0	1.00000	1.00000	1.00000	1.00000
5	0.99417	0.99543	0.99601	0.99601
10	0.99090	0.99427	0.99601	0.99601
20	0.98443	0.99200	0.99601	0.99601
30	0.97799	0.98973	0.99601	0.99601
40	0.97160	0.98743	0.99601	0.99601
50	0.96526	0.98522	0.99601	0.99601
60	0.95895	0.98297	0.99601	0.99601
70	0.95268	0.98072	0.99601	0.99601
80	0.94646	0.97848	0.99601	0.99601
90	0.94028	0.97625	0.99601	0.99601

Table 12. Effect of Failure rates on System Availability against Time for General Repair ($k = 7$ *and* $n = 10$)

Time	Availability $k = 7$ *and* $n = 10$ $v_1 = 0.001, v_2 = 0.002, v_3 = 0.003, v_c = 0.004$	Availability $k = 7$ *and* $n = 10$ $v_1 = 0, v_2 = 0.002, v_3 = 0.003, v_c = 0.004$	Availability $k = 7$ *and* $n = 10$ $v_1 = 0, v_2 = 0, v_3 = 0.003, v_c = 0.004$	Availability $k = 7$ *and* $n = 10$ $v_1 = 0, v_2 = 0, v_3 = 0, v_c = 0.004$
0	1.00000	1.00000	1.00000	1.00000
5	0.99445	0.99565	0.99601	0.99601
10	0.99163	0.99487	0.99601	0.99601
20	0.98606	0.99335	0.99601	0.99601
30	0.98051	0.99183	0.99601	0.99601
40	0.97500	0.99032	0.99601	0.99601
50	0.96952	0.98881	0.99601	0.99601
60	0.96407	0.98730	0.99601	0.99601
70	0.95866	0.98579	0.99601	0.99601
80	0.95327	0.98428	0.99601	0.99601
90	0.94791	0.98278	0.99601	0.99601

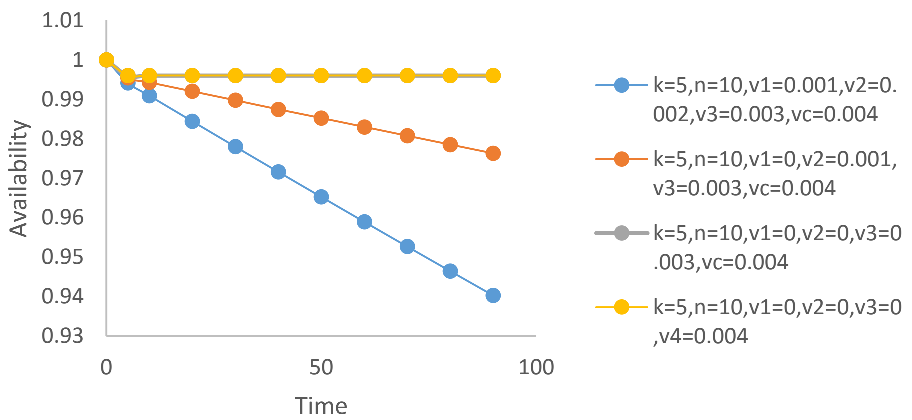

Figure 12. Effect of failure rates on availability against time for general repair

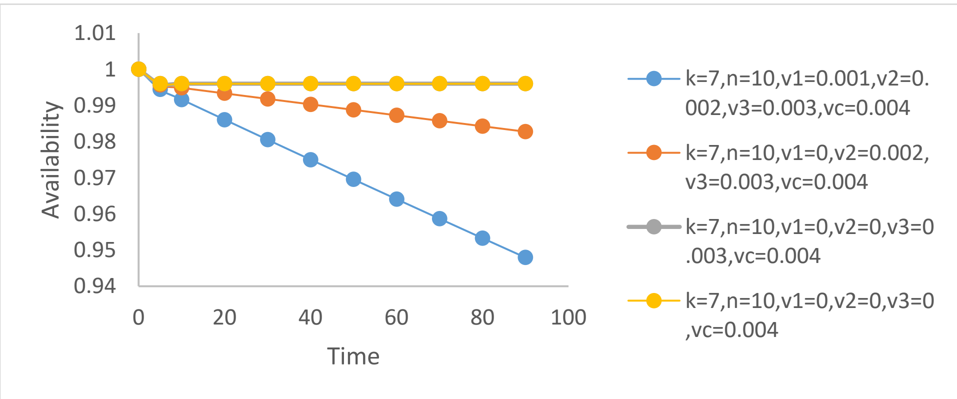

Figure 13. Effect of failure rates on availability against time for general repair

Table 13. Effect of Failure rates on System Availability against Time for General Repair ($k = 9$ and $n = 10$)

Time	Availability $k = 9$ and $n = 10$ $v_1 = 0.001, v_2 = 0.002, v_3 = 0.003, v_c = 0.004$	Availability $k = 9$ and $n = 10$ $v_1 = 0, v_2 = 0.002, v_3 = 0.003, v_c = 0.004$	Availability $k = 9$ and $n = 10$ $v_1 = 0, v_2 = 0, v_3 = 0.003, v_c = 0.004$	Availability $k = 9$ and $n = 10$ $v_1 = 0, v_2 = 0, v_3 = 0, v_c = 0.004$
0	1.00000	1.00000	1.00000	1.00000
5	0.99473	0.99589	0.99601	0.99601
10	0.99237	0.99548	0.99601	0.99601
20	0.98770	0.99472	0.99601	0.99601
30	0.98306	0.99396	0.99601	0.99601
40	0.97844	0.99320	0.99601	0.99601
50	0.97384	0.99244	0.99601	0.99601
60	0.96926	0.99168	0.99601	0.99601
70	0.96471	0.99092	0.99601	0.99601
80	0.96017	0.99016	0.99601	0.99601
90	0.95566	0.98940	0.99601	0.99601

Table 14. Effect of Failure rates on System Availability against Time for General Repair ($k = 5$ and $n = 15$)

Time	Availability $k = 5$ and $n = 15$ $v_1 = 0.001, v_2 = 0.002, v_3 = 0.003, v_c = 0.004$	Availability $k = 5$ and $n = 15$ $v_1 = 0, v_2 = 0.002, v_3 = 0.003, v_c = 0.004$	Availability $k = 5$ and $n = 15$ $v_1 = 0, v_2 = 0, v_3 = 0.003, v_c = 0.004$	Availability $k = 5 and$ $n = 15$ $v_1 = 0, v_2 = 0, v_3 = 0, v_c = 0.004$
0	1.00000	1.00000	1.00000	1.00000
5	0.99144	0.99428	0.99601	0.99601
10	0.98367	0.99120	0.99601	0.99601
20	0.96833	0.98511	0.99601	0.99601
30	0.95322	0.97905	0.99601	0.99601
40	0.93835	0.97302	0.99601	0.99601
50	0.92372	0.96701	0.99601	0.99601
60	0.90931	0.96109	0.99601	0.99601
70	0.89513	0.95517	0.99601	0.99601
80	0.88116	0.94930	0.99601	0.99601
90	0.86742	0.94345	0.99601	0.99601

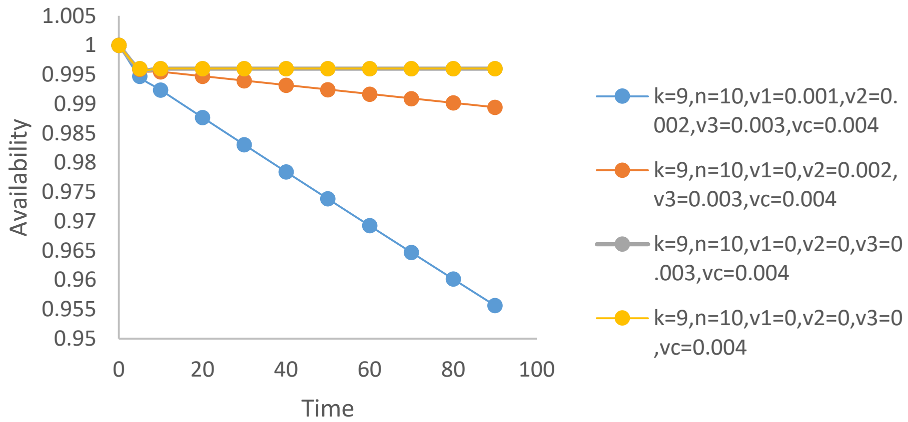

Figure 14. Effect of failure rates on availability against time for general repair

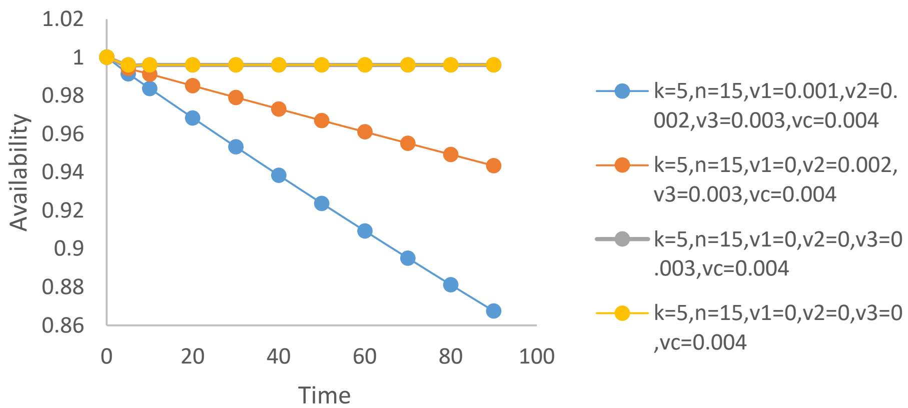

Figure 15. Effect of failure rates on availability against time for general repair

Table 15. Effect of Failure rates on System Availability against Time for General Repair ($k = 5$ *and* $n = 20$)

Time	Availability $k = 5$ *and* $n = 20$ $v_1 = 0.001, v_2 = 0.002, v_3 = 0.003, v_c = 0.004$	Availability $k = 5$ *and* $n = 20$ $v_1 = 0, v_2 = 0.002, v_3 = 0.003, v_c = 0.004$	Availability $k = 5$ *and* $n = 20$ $v_1 = 0, v_2 = 0, v_3 = 0.003, v_c = 0.004$	Availability $k = 5$*and* $n = 20$ $v_1 = 0, v_2 = 0, v_3 = 0, v_c = 0.004$
0	1.00000	1.00000	1.00000	1.00000
5	0.98764	0.99602	0.99601	0.99601
10	0.97371	0.99593	0.99601	0.99601
20	0.94644	0.99581	0.99601	0.99601
30	0.91994	0.99568	0.99601	0.99601
40	0.89417	0.99556	0.99601	0.99601
50	0.86913	0.99544	0.99601	0.99601
60	0.84479	0.99531	0.99601	0.99601
70	0.82113	0.99519	0.99601	0.99601
80	0.79813	0.99507	0.99601	0.99601
90	0.77578	0.99494	0.99601	0.99601

Table 16. Effect of Failure rates on System Availability against Time for General Repair ($k = 5$ *and* $n = 25$)

Time	Availability $k = 5$ *and* $n = 25$ $v_1 = 0.001, v_2 = 0.002, v_3 = 0.003, v_c = 0.004$	Availability $k = 5$ *and* $n = 25$ $v_1 = 0, v_2 = 0.002, v_3 = 0.003, v_c = 0.004$	Availability $k = 5$ *and* $n = 25$ $v_1 = 0, v_2 = 0, v_3 = 0.003, v_c = 0.004$	Availability $k = 5$*and* $n = 25$ $v_1 = 0, v_2 = 0, v_3 = 0, v_c = 0.004$
0	1.00000	1.00000	1.00000	1.00000
5	0.98286	0.99600	0.99601	0.99601
10	0.96132	0.99587	0.99601	0.99601
20	0.91961	0.99567	0.99601	0.99601
30	0.87972	0.99547	0.99601	0.99601
40	0.84156	0.99526	0.99601	0.99601
50	0.80505	0.99506	0.99601	0.99601
60	0.77013	0.99486	0.99601	0.99601
70	0.73672	0.99465	0.99601	0.99601
80	0.70477	0.99445	0.99601	0.99601
90	0.67419	0.99425	0.99601	0.99601

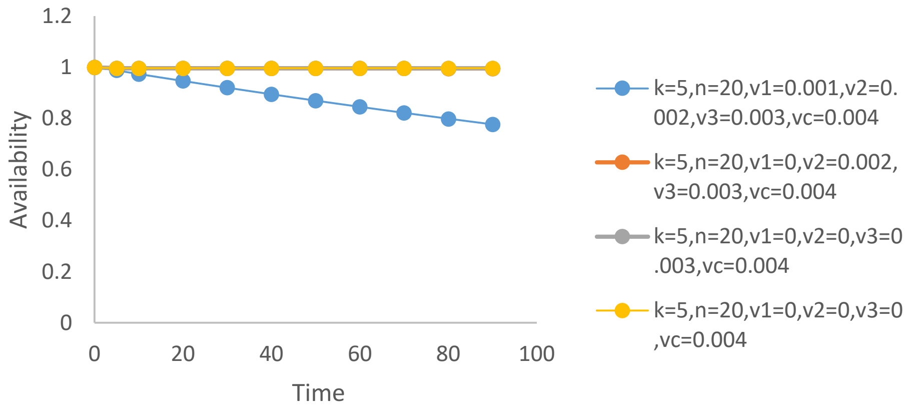

Figure 16. Effect of failure rates on availability against time for general repair

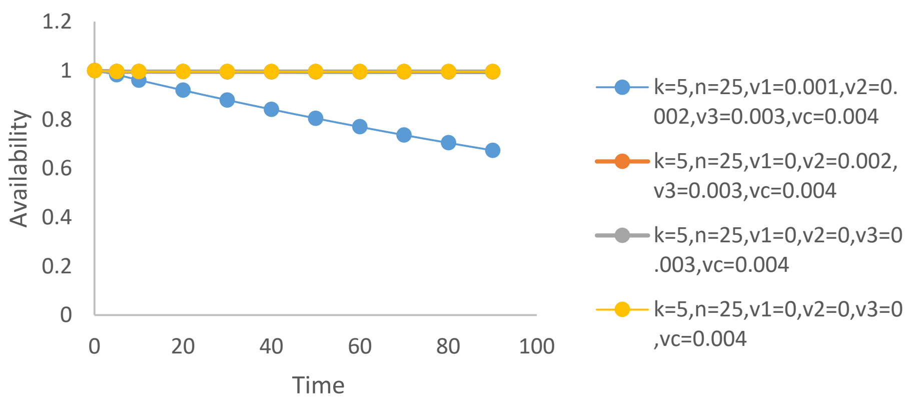

Figure 17. Effect of failure rates on availability against time for general repair

4.2. System Reliability Analysis

Following Chopra and Ram (2020), set all repairs to zero in equation (78)with the same failure rates, $v_1 = 0.001, v_2 = 0.002, v_3 = 0.003, v_c = 0.004$, and using inverse Laplace transform to get the following system reliability expression for $n = 5, k = 10$:

$$R(q) = 0.0001e^{-0.01q} + 0.0092e^{-0.012q} + 0.007142857143e^{-0.006q} + 0.00001153846154e^{-0.008q} + 2.5e^{-0.0026q} + 0.0024e^{-0.009q} + 0.000001e^{-0.004q} + 2.5e^{0.03q} - 4.030896775e^{-0.034q} + 0.012e^{-0.014q} + 0.00004137931034e^{-0.005q} \quad (92)$$

$$R(q) = 0.0002e^{-0.012q} - 1.5242e^{-0.0024q} + 2.5e^{-0.01q} + 0.024e^{-0.014q} \quad (93)$$

$$R(q) = 2.5e^{-0.01q} + 0.00005e^{0.008q} + 0.012e^{-0.09q} - 1.51205e^{-0.014q} \quad (94)$$

$$R(q) = 0.0001e^{-0.01q} + 0.0092e^{-0.012q} + 0.007142857143e^{-0.006q} + 0.00001153846154e^{-0.008q} + 2.5e^{0.026q} + 0.0024e^{-0.009q} + 0.000001e^{0.04q} + 2.5e^{-0.03q} - 4.030896775e^{-0.034q} + 0.012e^{-0.014q} + 0.00004137931034e^{0.005q} \quad (95)$$

$$R(q) = 0.01212e^{-0.01q} + 0.01111111111e^{-0.012q} + 2.5e^{0.022q} + 0.002448e^{0.005q} + 2.5e^{0.026q} + 0.00001269230769e^{-0.004q} - 4.034134228e^{-0.03q} + 0.008333333333e^{0.06q} + 0.0001090909091e^{-0.008q} \quad (96)$$

Using (92)-(96), one may get different values of reliability for various values of time $t = 0,510,20,30,40,50,60,70,80,90$ units of time as presented in table 17 and figure 18.

Table 17: System Reliability Analysis against Time

Time	Reliability $v_1 = 0.001, v_2 = 0.002, v_3 = 0.003, v_c = 0.004$, $k = 5$, and $n = 10$	Reliability $v_1 = 0, v_2 = 0.002, v_3 = 0.003, v_c = 0.004$, $k = 5$, and $n = 10$	Reliability $v_1 = 0.001, v_2 = 0, v_3 = 0.003, v_c = 0.004$, $k = 5$, and $n = 10$	Reliability $v_1 = 0.001, v_2 = 0.002, v_3 = 0, v_c = 0.004$, $k = 5$, and $n = 10$	Reliability $v_1 = 0.001, v_2 = 0.002, v_3 = 0.003, v_c = 0$, $k = 5$, and $n = 10$
0	1.00000	1.00000	1.00000	1.00000	1.00000
5	0.97551	0.97851	0.97977	0.97551	0.99520
10	0.93826	0.95242	0.95859	0.93826	0.97648
20	0.84099	0.89052	0.91411	0.84099	0.91079
30	0.73114	0.82096	0.86776	0.73114	0.82388
40	0.62199	0.74846	0.82051	0.62199	0.72919
50	0.52071	0.67627	0.77315	0.52071	0.63498
60	0.43059	0.60657	0.72629	0.43059	0.54611
70	0.35268	0.54072	0.68039	0.35268	0.46510
80	0.28669	0.47954	0.63584	0.28669	0.39303
90	0.23167	0.42342	0.59289	0.23167	0.33006

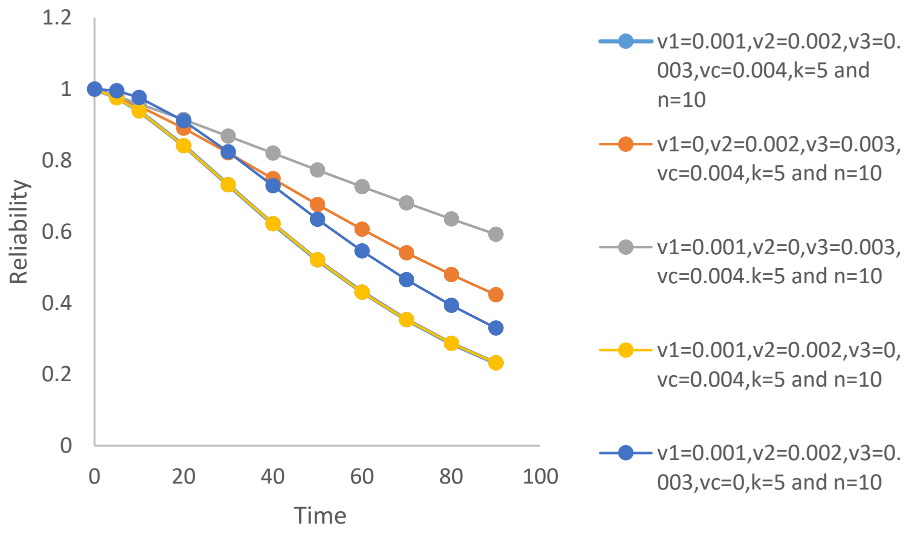

Figure 18. Reliability against time for general repair

4.3. Mean Time To Failure (MTTF) Analysis

System MTTF can be attained by setting all the repair rates to zero in equation (79) and finding the limit of the expression as s goes to zero. The expression below gives the MTTF of the system:

$$MTTF = \lim_{s\to 0}\overline{F_{up}}(s) = \frac{1}{G(s)}\left[1+\left\{\frac{nv_1}{y_1v_1+nv_2+v_c}\right\}+\left\{\frac{ny_1v_1^2}{y_2v_1+v_c}\right\}+\left\{\frac{ny_1y_2v_1^3}{y_3v_1+v_c}\right\}+\left\{\frac{nv_2}{y_1v_2+nv_1+v_c}\right\}+\left\{\frac{ny_1v_2^2}{y_2v_2+v_c}\right\}+\left\{\frac{ny_1y_2v_2^3}{y_3v_2+v_c}\right\}+\left\{\frac{n^2v_2}{y_1}\right\}+\left\{\frac{n^2y_1v_1v_2}{y_2}\right\}+\left\{\frac{n^2y_1y_2v_1^2v_2}{y_3}\right\}+\left\{\frac{n^2v_1}{y_1}\right\}+\left\{\frac{n^2y_1v_1v_2}{y_2}\right\}+\left\{\frac{n^2y_1y_2v_1v_2^2}{y_3}\right\}\right] \quad (97)$$

For the simulation of MTTF with respect to v_1, v_1 is varied between 0.001 to 0.10 while $v_2 = 0.002$ and $v_c = 0.004$ are fixed, for MTTF with respect to v_2, v_2 is varied between 0.001 to 0.10 while $v_1 = 0.001$ and $v_c = 0.004$ are fixed, MTTF with respect to v_c, v_c is varied between 0.001 to 0.10 while $v_2 = 0.002$ and $v_2 = 0.002$ are fixed respectively.in (97). one may obtain Table 18 and Figure 19showing variation of MTTF with respect to v_1, v_2, v_3 andv_c.

Table 18. Variation of MTTF with respect to Failure rates

Failure rate	$MTTF$ v_1	$MTTF$ v_2	$MTTF$ v_3	$MTTF$ v_c
0.001	63.51234	84.73093	63.51234	74.72454
0.002	49.92024	63.51234	63.51234	70.59374
0.003	41.87423	51.52871	63.51234	66.87711
0.004	36.38844	43.61618	63.51234	63.51234
0.005	32.34359	37.94729	63.51234	60.45055
0.006	29.21066	33.66924	63.51234	57.65227
0.007	26.70021	30.31992	63.51234	55.08501
0.008	24.63753	27.62407	63.51234	52.72163
0.009	22.90961	25.40645	63.51234	50.53919
0.010	21.43944	23.54978	63.51234	48.51814

Table 19. Computation of System Sensitivity against failure rate parameters

Failure rate	Sensitivity v_1	Sensitivity v_2	Sensitivity v_3	Sensitivity v_c
0.001	-18638.94025	-30142.56243	0.00000	-4363.23684
0.002	-10024.14288	-15180.46926	0.00000	-3911.93608
0.003	-6496.229164	-9504.596287	0.00000	-3531.60244
0.004	-4646.290805	-6598.190650	0.00000	-3206.01810
0.005	-3526.805670	-4873.889088	0.00000	-2924.10744
0.006	-2785.484806	-3756.273159	0.00000	-2677.84646
0.007	-2263.731385	-2986.922582	0.00000	-2461.17709
0.008	-1880.055565	-2433.332865	0.00000	-2269.38745
0.009	-1588.414622	-2021.111595	0.00000	-2098.73048
0.010	-1360.889373	-1705.613029	0.00000	-1946.17509

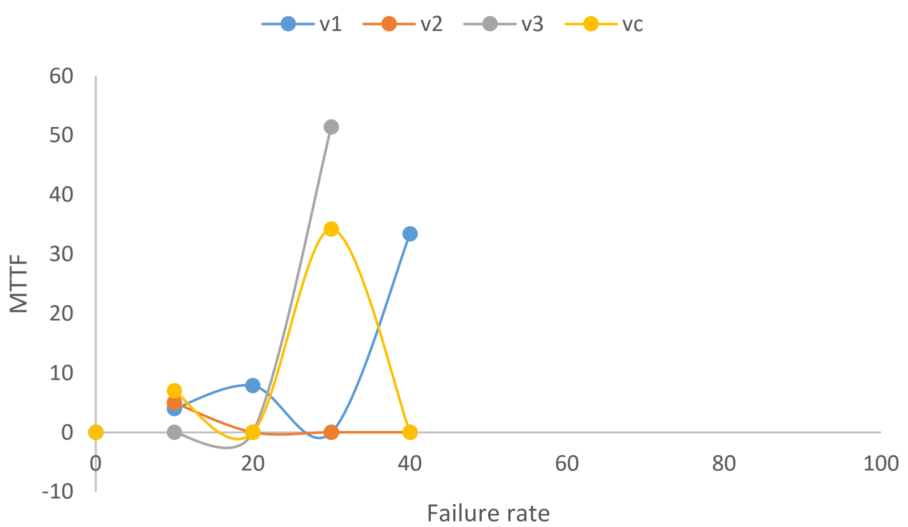

Figure 19. Mean time to failure against failure rate

4.4. System Sensitivity Analysis

To gauge system sensitivity, equation (97)'s partial derivative of MTTF with regard to failure rates is used. Applying the set of parameters as $v_1 = 0.001, v_2 = 0.002, v_3 = 0.003, and v_c = 0.004$ in the partial differentiation of MTTF will yield the system sensitivity as indicated in table 19 and the related graph in figure 20.

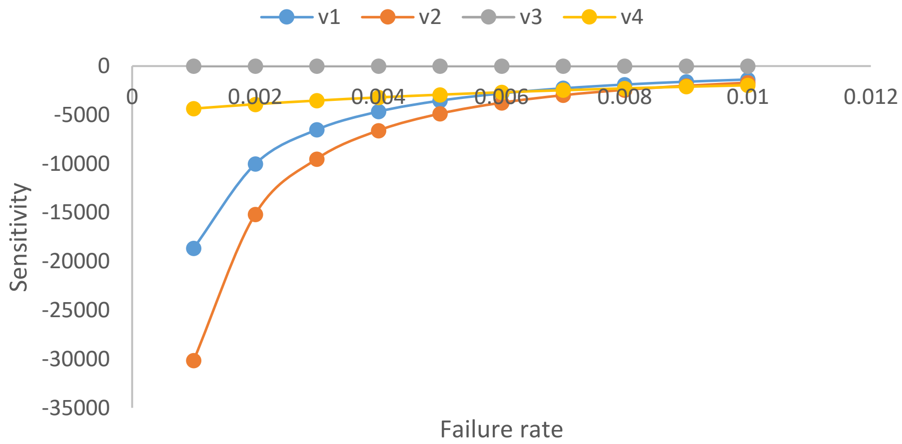

Figure 20: MTTF sensitivity failure rates

4.5. Cost Benefit/Benefit Function

If the service facility is always open, the formula below can be used to predict the expected profit for the range [0,1).

$$E_p(t) = H_2 \int_0^t F_{up}(t)dt - H_2 t. \quad (98)$$

Where H_1 and H_2 in the range [0, t) represent the revenue generated and service cost per unit time.

4.5.1. Expected Profit for Copula Repair

Supposing that the failure rates of the system are given as follows: $v_1 = 0.001, v_2 = 0.002, v_3 = 0.003$, one can obtain equation (99) by combining equations (80) and (98) as:

$$E_p(q) = \left\{ \begin{array}{l} 1.31618078510^{-1.004q} + 1.16887228510e^{-1.008q} + \\ 0.00002257401742e^{-1.009q} + 0.00007219922840e^{-1.014q} \\ +0.00008300200201e^{-1.006q} - 0.000005655223134e^{-2.722494532q} \\ +0.001162473735e^{-1.056605286q} + 0.000005238282313e^{-1.028543017q} \\ -1521.601946e^{-0.00065716478 93q} + 0.00006736972585e^{-1.012q} \\ +8.69601951910e^{-1.01q} + 5.12476837110e^{-1.005q} \\ +1521.601097 \end{array} \right\} - H_2 q \quad (99)$$

Table 20 and figure 21 are obtained for Copula repairs by using different values of the time variable, such as $t = 0,5,10,20,30,40,50,60,70,80,90$, and by taking the inverse Laplace transform of equation (99) with $C_1 = 1$ and $C_1 = 0.6,0.5,0.4,0.3,0.2,0.1$, respectively

Table 20. Expected Profit for Copula Repair

Time	$E_p(t)$ $H_2 = 0.6, k = 5$ and $n = 10$	$E_p(t)$ $H_2 = 0.5, k = 5$ and $n = 10$	$E_p(t)$ $H_2 = 0.4, k = 5$ and $n = 10$	$E_p(t)$ $H_2 = 0.3, k = 5$ and $n = 10$	$E_p(t)$ $H_2 = 0.2, k = 5$ and $n = 10$	$E_p(t)$ $H_2 = 0.1, k = 5$ and $n = 10$
0	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
5	1.99067	2.49067	2.99067	3.49067	3.99067	4.49067
10	3.96580	4.96580	5.96580	6.96580	7.96580	8.96580
20	7.86717	9.86717	11.86717	13.86717	15.86717	17.86717
30	11.70368	14.70368	17.70368	20.70368	23.70368	26.70368
40	15.47576	19.47576	23.47576	27.47576	31.47576	35.47576
50	19.18383	24.18383	29.18383	34.18383	39.18383	44.18383
60	22.82831	28.82831	34.82831	40.82831	46.82831	52.82831
70	26.40962	33.40962	40.40962	47.40962	54.40962	61.40962
80	29.92817	37.92817	45.92817	53.92817	61.92817	69.92817
90	33.38438	42.38438	51.38438	60.38438	69.38438	78.38438

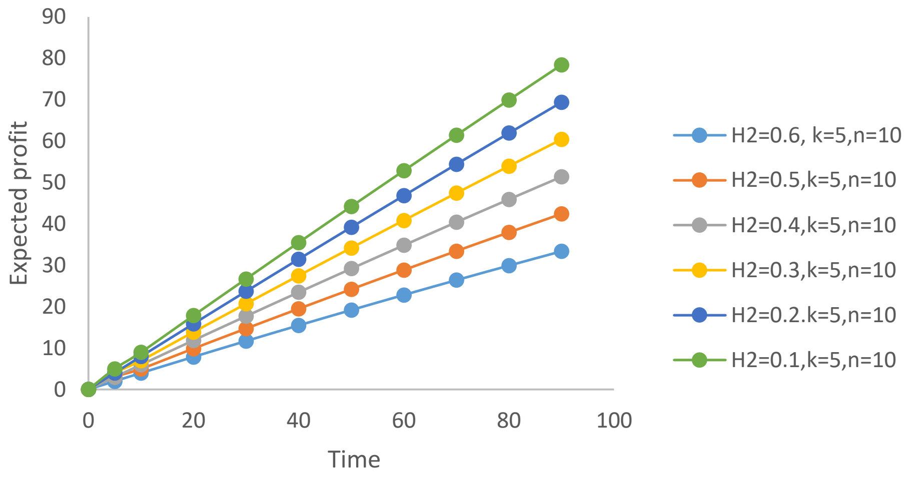

Figure 21. Expected profit against time for copularepair

4.5.2. Expected Profit for General Repair

For General repairs, the expected profit function is obtained by using equations (79) and () and the failure rate parameters of equation (99) and is given by:

$$E_p(q)=\left\{\begin{array}{l} 2.42261358010e^{-1.004q}+1.47008476010e^{1.008q} \\ +0.00002740634340e^{-1.009q}+0.00007949717781e^{-1.014q} \\ +0.0001168220524e^{-1.006q}+0.00007637403477e^{1.012q+} \\ 0.000001026950771e^{1.01q}-0.0008781644108e^{-1.058869190q} \\ -0.00001341856965e^{-1.028562257q}-0.0001941737209e^{-1.00913048q} \\ 1521.598564e^{-0.0006555053320q}+7.96479379310e^{-1.005q} \\ +1521.601095 \end{array}\right\}-H_2q \quad (100)$$

Table 21 and figure 22 are obtained for General repairs by using various values of the time variable, such as $t=0,5,10,20,30,40,50,60,70,80,90$, and by taking the inverse Laplace transform of equation (100) with $C_1=1$ and $C_1=0.6,0.5,0.4,0.3,0.2,0.1$, respectively.

Table 21. Expected Profit for General Repair

Time	$E_p(t)$ $H_2=0.6,k=5$ and $n=10$	$E_p(t)$ $H_2=0.5,k=5$ and $n=10$	$E_p(t)$ $H_2=0.4,k=5$ and $n=10$	$E_p(t)$ $H_2=0.3,k=5$ and $n=10$	$E_p(t)$ $H_2=0.2,k=5$ and $n=10$	$E_p(t)$ $H_2=0.1,k=5$ and $n=10$
0	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
5	1.98143	2.48143	2.98143	3.48143	3.98143	4.48143
10	3.94407	4.94407	5.94407	6.94407	7.94407	8.94407
20	7.82066	9.82066	11.82066	13.82066	15.82066	17.82066
30	11.63271	14.63271	17.63271	20.63271	23.63271	26.63271
40	15.38066	19.38066	23.38066	27.38066	31.38066	35.38066
50	19.06492	24.06492	29.06492	34.06492	39.06492	44.06492
60	22.68591	28.68591	34.68591	40.68590	46.68591	52.68591
70	26.24403	33.24403	40.24403	47.24403	54.24403	61.24403
80	29.73971	37.73971	45.73971	53.73971	61.73971	69.73971
90	33.17335	42.17335	51.17335	60.17335	69.17335	78.17335

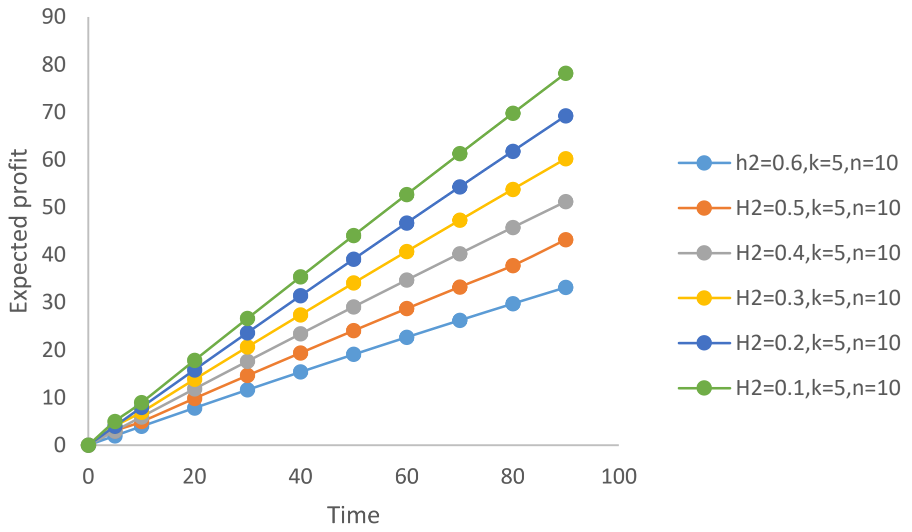

Figure 22. Expected profit against time for general repair

5. Discussion of Results

The performance of the system is referred to as availability when repairs are provided to the malfunctioning components/units. To distinguish between the two, we have given the system's availability in two different ways: when repairs come after Copula repairs and when repairs come after General repairs. Additionally, we have investigated availability in two scenarios: $k = 5,7,9, n = 10$ and $k = 5, n = 10,15,20,25$, when repairs adhere to Copula and General repair policy. Furthermore, it is essential to consider how each failure rate impacts system availability when the value of n is fixed and k values are varied as well as when the value of k is fixed and $n$values are modified for both Copula and General repairs.Table 1 and figure 2 provide information on system availability when repairs are carried out using the Copula repair policy and the values of k are varied as $k = 5,7,9$ while n is maintained constant at $n = 10$. This table and figure show that, in general, the system's availability declines as the value of time t increases. This demonstrates how the graphical representation may be used to predict system performance with accuracy. Furthermore, table 1 and figure 2 show that the operational availability does, however, grow as the value of k increases and n remains constant. But when k is kept constant at $k = 5$ and n is altered as $n = 15,20,25$, table 2 and figure 3 indicate the availability of the system for Copula repair. We can see from this table and figure that, on average, operational availability declines as time goes on. However, when n increases and k stays constant, operational availability diminishes. Likewise, when the repairs follow General repair and$k$ is varied as $k = 5,7,9$, while n is kept constant at $n = 10$, the information on system availability is provided in table 9 and figure 10. Similar to the Copula repair scenario, the operational availability of the system increases generally with an increase in the value of time t and also, as the value of k grows with n fixed, the operational availability increases. Similarly, when the repairs obey General repair and k is kept constant $k = 5$ while n is modified as $n = 15,20,25$, table 10 and figure 11 offer the information on system availability which decreases with an increase in the value of time t. But as n grows, the operational availability decreases. However, both scenarios have substantially better operational availability for Copula repair than they do for General maintenance. Intriguingly, the operational availability for both Copula and General repair improves as the value of k rises, whereas the operational availability for both types of repairs decreases as the value of n rises. This is premised on the physical fact that a system consisting of k out of n will function if and only if k or more of the components function. These analyses suggest that the system availability can be maximized by using Copula repair and more functional units. Furthermore, tables3,4,5,6,7,8,11,12,13,14,15,16 and their associated figures4,5,6,7,8,9,12,13,14,15,16,17 depict the impact of each failure rate on system availability for both Copula and General repairs when n is fixed and k is varied as well as when k is fixed and n is varied.These tables and graphs show that the system is more available when the failurev_1 and v_2 are zeros for all the situations. The significance of failure rates v_3 and v_c is explained in this analysis.

Reliability is the probability that a system will continue to work properly without a repair. Here, consideration is also given to how each failure rate affects system reliability and the results are shown in table 17 and figure 18. The table and figure clearly show that in general, system reliability decreases with passing time. This is a direct result of inadequate repair. However, we limit each failure rate to zero in order to see the effect it has on system reliability. We observe from table 17 and figure 18 that the reliability value at $v_3 = 0$ is equal to the cumulative impact of all failure rates on system reliability, which is incredibly low. The importance of any failure rate on system reliability is shown in the value of reliability, i.e., the lower the reliability value, the more critical the failure rate. When the failure rate v_3 is under control, the system can be as reliable as possible. Maintenance engineers should, therefore, device a method to prevent hub failure.

With the aid of crucial failure metric known as Mean Time To Failure (MTTF), maintenance mangers and system engineers can anticipate how long non-repairable systems will function before failing. When other parameters are kept constant, the system's MTTF is shown in table 18 and figure 19 as a function of variations in $v_{1,}v_2, v_3,$ and v_c, respectively. The MTTF of the system decreases as v_1, v_2 and v_c change, but remains constant as v_3 changes. It is interesting to see that the MTTF with respect to v_3 is continuous, indicating that the hub must be in operation at all times. This analysis provides support for the hypothesis that failure rate v_3 has a greater impact on system reliability.

An important aspect of engineering and scientific analysis is the assessment of sensitivities. Sensitivity analysis gives data on the relative significance of model input variables, and they help with both model validation and optimization. The results of the sensitivity analysis highlight the importance of each component. Table 19 and figure 20 summarize the results of sensitivity analysis studied in this research. This figure and table show that the system is sensitive to the failure rate v_3. The sensitivity results reveal that the system effectiveness can be optimized by controlling the failure rate v_3. The influence of failure rate v_3 on system reliability has once again been proven by this sensitivity analysis.

Cost benefit analysis is a generic technique that is widely employed in engineering to evaluate choices/decisions and systems as well as to calculate the value of intangible assets. Cost benefit analysis is used in many industries to estimate the maximum and future worth of a design or system. Generally speaking, an industrial manager will frequently attempt to boost the industry's profit since earnings are determined by both growing or rising revenue and reducing operational costs. Managers typically choose this strategy because of its apparent significance in terms of increasing profitability. When the service cost, H_2 is modified as 0.6,0.5,0.4,0.3,0.2,0.1, respectively, and the income generated per unit of time, H_1, is fixed at $1, k = 5,$ and $n = 10$. Table 20 and figure 21 show the projected profit from the system when repairs are made after Copula repair, while table 21 and figure 22 show the expected profit after General repair. Table 20 and figure 21 show that over time, the projected profit increases as service costs, H_2, fall. The projected profit is often higher when comparing low service $(H_2 = 0.1)$ cost to high service cost $(H_2 = 0.6)$.Hence, maintenance mangers and system engineers should select the optimal service fee based on the anticipated profit level. The same Copula repair outcome is shown in table 21 and figure 22 for General repair. Though the predicted profit for Copula repair is substantially higher than that for General repair. This analysis supports the claim or premise that Copula repair increases system availability more than General repair.

6. Conclusion

In the present study, the functional behavior of a serial system employing k-out of-n configuration in each subsystem is discussed. The study examined the performance of serial sysem with k-out-of-n units using the features of Gumbel-Hougaard family Copula. The expressions for the system characteristics, such as availability, reliability, mean time to failure (MTTF), MTTF sensitivity, and predicted profit were obtained and validated through numerical experiments. The impact of the different parameters governing system was examined. Tables and figures are used to present the findings. Based on these findings, the following observations were reached:

1. It was discovered that system availability and predicted profit (cost) for the system rise when Copula repair is employed. Therefore, this repair technique is more effective in raising predicted profit and availability. This has given engineers a new reason to accept copula-based multi-dimension repair.
2. Furthermore, it was demonstrated that controller failure reduces system reliability, mean time to failure and availability, cost, and overall system performance.
3. Furthermore, it was found that low service costs resulted in higher-than-expected profits for the system. Maintenance mangers and system engineers should select the ideal service fee based on the expected profit level.
4. The study suggests that the management should employ additional controller working in parallel or standby with the main controller. This will reduce complete system failure due to controller.
5. Also, it was shown that as the value of k rises, the operational availability for both Copula and General repairs gets better. This analysis suggests that more working units can be invoked.

This research lays the groundwork for maintenance staff and system architects to identify the best types of repairs and system configurations, as well as skilled controllers, to enhance overall efficiency and revenue generation. Furthermore, modifying the models/results described in this work will enable management to avoid incorrect reliability evaluations and incorrect decision-making, resulting in wasteful spendings.The current research can extend to address the system with load sharing subsystems, inspection strategies. This topic will be explored more in our future work.

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