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Improving the COPRAS Multicriteria Group Decision-Making Method for Selecting a Sustainable Supplier Using Intuitionistic and Fuzzy Type 2 Sets

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Abstract

Recent years have seen an increase in the importance of assessing the environmental and social impacts of industrial product supply chains, leading to the introduction of the concept of supplier sustainability, which entails meeting all suppliers' economic, environmental, and social needs. Typically, supplier selection decisions are based on expert opinions presented as supplier scores. Experts may not be equally familiar with all aspects of suppliers' economic, social, and environmental attributes when evaluating sustainability metrics. The purpose of this paper is to present a new approach for selecting a sustainable supplier using the COPRAS (Complex Proportional Assessment) multicriteria group decision-making process. Suppliers' scores and the weights of each criterion are expressed verbally by experts, by using linguistic variables. Then the linguistic terms are transformed into equivalent type 2 fuzzy numbers. Using fuzzy type 2 numbers facilitates aggregating experts' opinions during group decision-making. The proposed approach assumes that the problem analyst, in addition to collecting the suppliers' scores given by the experts, also determines a degree of expertise for each expert in each criterion and aggregates the data by the COPRAS method to determine the optimal decision. The analyst's verbal variables to represent his view on the validity of each expert are then converted into intuitionistic fuzzy numbers. Intuitionistic fuzzy numbers provide the possibility of presenting uncertainty and doubt from the analysts' point of view. This paper illustrates the application of the model by presenting an example and discussing its results.

Keywords: Type II fuzzy set, Intuitionistic fuzzy set, Multi-criteria decision making, Fuzzy COPRAS, Sustainable supplier selection.

1. Introduction

Choosing a supplier is one of the most critical decisions for all organizations, and in many cases, they use multicriteria decision-making (MCDM) methods based on economic criteria. In recent years, due to increasing global warming and other adverse effects of human activities on nature, the importance of considering environmental impacts in all aspects of the industrial products supply chain has increased, including the issue of supplier selection. The sustainable supplier selection problem(SSSP)is a complex decision, and often the inconsistency of sustainability metrics and organizational goals adds to its complexity. Decisions are usually made based on expert ratings of different suppliers. During SSS, all the required data are not always available with certainty, and there is inevitable ambiguity and uncertainty in some data. Experts use linguistic variables to express the vagueness of their opinions. The fuzzy theory allows converting linguistic terms into type 1,type 2,or intuitionistic fuzzy numbers. In addition, when evaluating sustainability metrics, it is essential to note that these experts may not be equally knowledgeable in all three economic, social, and environmental fields. This paper presents a new approach to selecting a sustainable supplier using the COPRAS method. The COPRAS method is a ranking technique that is simple, very practical, and powerful, and its calculations do not require complex mathematical operations. This method considers the effect of maximizing favorable indicators and minimizing unfavorable indicators on the evaluation results separately, and by considering the conflicting conditions of the criteria and based on the weight of the criteria, it compares the proposals and determines the priorities. The proposed approach assumes that in addition to the experts, there is an analyst who, besides collecting the scores given by experts, also evaluates these experts' knowledge and aggregates the data to determine the optimal decision. The second section presents the literature and the background of previously conducted research on selecting sustainable suppliers. The third section describes the research problem and the steps to solve it according to the proposed method. The fourth section describes the proposed approach's application and the process by solving a numerical example and analyzing the results. The fifth section of the paper discusses the results and concludes the research findings.

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2. Literature review and research background

Fuzzy theory-based approaches are simple and require little exact data. The membership value in a type-1 fuzzy set is a real value in the range of 0 to 1.The type 2 fuzzy set expresses an additional aspect of uncertainty by assigning secondary values to the degrees of membership of a regular fuzzy number.Type-2 fuzzy sets extend conventional (type-1) fuzzy sets by defining primary and secondary membership values.Using type 2 fuzzy sets to solve problems requires more calculations; however, it provides more significant degrees of freedom and flexibility.

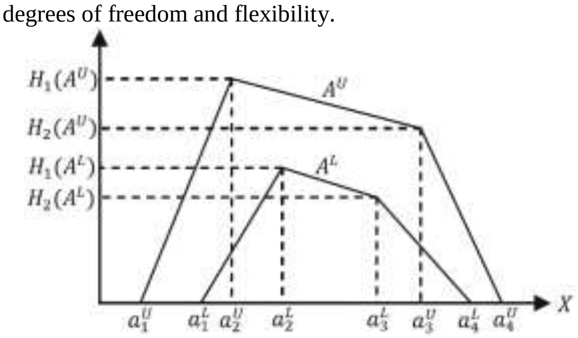

Figure 1. The trapezoids type 2fuzzy sets[1]

As Figure 2 illustrates, a trapezoidal interval type-2 fuzzy number (TI2FN) such as$\tilde{\tilde{A}}$, has upper and lower membership functions, which are trapezoidal fuzzy numbers. It is shown in the following format [1]:

$$\tilde{\tilde{A}} = \left(\tilde{A}^U, \tilde{A}^L\right) = ((a_1^U, a_2^U, a_3^U, a_4^U; H_1(\tilde{A}^U), H_2(\tilde{A}^U)), ((a_1^L, a_2^L, a_3^L, a_4^L; H_1(\tilde{A}^L), H_2(\tilde{A}^L)))$$

The summation of two TI2FNs, $\tilde{\tilde{A}}_1$ and $\tilde{\tilde{A}}_2$ is:

$$\tilde{\tilde{A}}_1 \oplus \tilde{\tilde{A}}_2 = \left(\begin{pmatrix} a_{11}^U + a_{21}^U, a_{12}^U + a_{22}^U, a_{13}^U + a_{23}^U, a_{14}^U + a_{24}^U; \\ \min\left(H_1(\tilde{A}_1^U), H_1(\tilde{A}_2^U)\right), \\ min\left(H_2(\tilde{A}_1^U), H_2(\tilde{A}_2^U)\right) \end{pmatrix}, \begin{pmatrix} a_{11}^L + a_{21}^L, a_{12}^L + a_{22}^L, a_{13}^L + a_{23}^L, a_{14}^L + a_{24}^L; \\ \min\left(H_1(\tilde{A}_1^L), H_1(\tilde{A}_2^L)\right), \\ min\left(H_2(\tilde{A}_1^L), H_2(\tilde{A}_2^L)\right) \end{pmatrix}\right) \quad (1)$$

Where:

$$\tilde{\tilde{A}}_1 = \left(\tilde{A}_1^U, \tilde{A}_1^l\right) = ((a_{11}^U, a_{12}^U, a_{13}^U, a_{14}^U; H_1(\tilde{A}_1^U), H_2(\tilde{A}_1^U)), ((a_{11}^L, a_{12}^L, a_{13}^L, a_{14}^L; H_1(\tilde{A}_1^l), H_2(\tilde{A}_1^l))) \quad (2)$$

and:

$$\tilde{\tilde{A}}_2 = \left(\tilde{A}_2^U, \tilde{A}_2^l\right) = ((a_{21}^U, a_{22}^U, a_{23}^U, a_{24}^U; H_1(\tilde{A}_2^U), H_2(\tilde{A}_2^U)), ((a_{21}^L, a_{22}^L, a_{23}^L, a_{24}^L; H_1(\tilde{A}_2^l), H_2(\tilde{A}_2^l))) \quad (3)$$

Also, the result of the division of a TI2FN by a nonzero number k is:

$$\tilde{\tilde{A}}/k = \begin{cases} ((a_1^U/k, a_2^U/k, a_3^U/k, a_4^U/k; H_1(\tilde{A}^U), H_2(\tilde{A}^U)), \\ ((a_1^L/k, a_2^L/k, a_3^L/k, a_4^L/k; H_1(\tilde{A}^L), H_2(\tilde{A}^L)) & if\ k > 0 \\ ((a_4^U/k, a_3^U/k, a_2^U/k, a_1^U/k; H_1(\tilde{A}^U), H_2(\tilde{A}^U)), \\ ((a_4^L/k, a_3^L/k, a_2^L/k, a_1^L/k; H_1(\tilde{A}^L), H_2(\tilde{A}^L)) & if\ k < 0 \end{cases} \quad (4)$$

Intuitionistic fuzzy sets (IFS) are introduced to extend the flexibility of fuzzy sets. IFSs are for describing vague and inaccurate information and dealing with uncertainty and ambiguity in the decision-making process. When the decision-makers express their opinion through IFSs, they do not consider the degree of non-membership to complement the degree of membership and regard it as doubtful. Using IFSs in complex MCDM problems attracts much attention due to possible imprecision and fuzziness in the real-world [2]. According to Chang et al., the use of IFS produces more accurate results in real situations[3]. Shahrokhi et al. used IFSs to describe incomplete and inaccurate information better and used two customer satisfaction indicators and flexibility in linear programming to determine suppliers and order quantities[4]. They calculated the flexibility index of the suppliers' production based on the additional production volume and variety of the product and showed their responsiveness when the buyer's needs changed. They used IFS to express satisfaction with the "supplier" with three factors: quality, price, and time. Hashemi et al. selected the most appropriate set of suppliers by considering all three sustainability objectives in highway construction with a group decision-making approach[5].Tronnebati et al. compare the difference between the concept of green supply chain management and sustainable supply chain management in the literature [6].The green supply chain activities improve environmental health by reducing its impact on many areas of life, such as reducing consumption of food, energy, water, and proper waste disposal. The sustainable supply chain is a more complete approach and in addition to the items considered in the green supply chain, it is designed by taking into account economic, environmental, and social considerations, to ensure the continued activity of the industry in the future in a healthy and sustainable environment. Umrosman solved an SSSP with a multi-objective model using uncertain targets based on interval type-2 fuzzy numbers[7]. Jraisat et al. identified two types of key issues affecting participation as drivers of internal and external focused participation and investigated how they influence the implementation of sustainable value chains[8]. Chiu et al. consider an integrated seller-buyer batch manufacturing problem with outsourcing, rework, machine breakdowns, and multiple deliveries to facilitate better decision-making and help companies increase competitive advantages[9]. Memari et al. presented an intuitionistic fuzzy TOPSIS method for selecting a sustainable auto spare parts supplier with nine main criteria and 30 sub-criteria for the manufacturer[10].Reference [11] considered six factors to build a flood risk map in Wadi Al-Mafraq using Geographic Information System (GIS) and Multi-criteria Design Analysis (MCDA). Reference [12] used both the Analytical Hierarchy Process (AHP) alone and the combination of AHP and the technique for order of preference by similarity to ideal solution (AHP-TOPSIS) to develop a methodology to choose the most competitive waste-to-energy technology, by evaluating three main criteria: environmental, technical, and socioeconomic; with three sub-criteria under each main criterion. To better model human thinking, Esser et al. extracted relative weights in SSCM, with the fuzzy best-worst method (F-BWM). After that, they integrated the traditional CoCoSo solution method with normalized weighted average functions and improved the normalized Bonferroni weighted geometric mean to select the most suitable supplier in the supply chain [13] .Esser used extended AHP under a 2-interval type fuzzy environment model (IT2FAHP) to better deal with ambiguity and solve supplier selection problems (SSPs) by considering green concepts [14]. Esser et al evaluated fifteen known crypto currencies with the highest market value based on sixteen factors and an intuitive fuzzy set-based method including evaluation based on distance from average solution (EDAS), real comparative analysis of multiple ideals. feature (MAIRCA), and Measure Options and Ranking Based on Compromise Solution (MARCOS), to provide a robust group decision-making tool in the field of digital currency selection [15].Bali et al presented “Intuitive Fuzzy Sets (IFS) along with the TOPSIS method to evaluate and select the best COTS vendor in a group decision environment considering reliability and delivery[16]. Darfshan et al. project a critical path according to the criteria of time, cost, risk, and quality under type 2 fuzzy numbers, through a new decision-making model, that is, MABACODAS, under parametric fuzzy values, to quantify supply risks in activities and presented project objectives. It is determined first. MABACODAS consists of two parts [17]. Gharoui et al. used pentagonal intuitionistic fuzzy number (PIFN) methods with AHP and order priority method with similarity to ideal solution (FTOPSIS) methods to rank cloud service providers (CSPs) [18].Manickamet al. presented a two-machine manufacturing scheme to investigate the optimal manufacturing-delivery policy for a two-stage multi-item system with a common part, postponement policy, and product quality assurance (including product screening, scrap, and rework)[19].Sustainable supplier selection is an MCDM problem with qualitative and qualitative attributes, and so far, supplier selection studies have used various methods. In recent years, using the COPRAS method as one of the multicriteria decision-making methods has increased SSPs due to the simplicity of calculation, complete ranking of options, and consideration of positive and negative criteria. Figure 1 illustrates the steps of this method. The original version of the COPRAS method was developed for decision-making in definite situations, but since uncertainty is an inevitable feature, this method was later advanced for uncertain conditions. The capability of this method in group decision-making and using fuzzy theory increases its flexibility and powers it in using inaccurate information and expert opinion. A new method is presented by [20] to compare alternatives and factors or criteria from fuzzy pairwise judgments. The linguistic evaluations or assessments express the relative importance of pairs which are quantified in the form of trapezoidal fuzzy numbers. The problem of finding components of the priority vector is formulated in an optimization model and is solved using a genetic algorithm. They presented an evaluation of selection criteria and a method for crane selection using simple additive weighting and fuzzy simple additive weighting methods[21].Hassan et al. prioritized the construction methods used in Syria and determined how to choose and the role of renewable energy criteria in the selection of the reconstruction plan using the AHP method[22]. Jreissat et al. (2019) identified five types of issues that influence information sharing in collaboration for sustainable relationships[23].

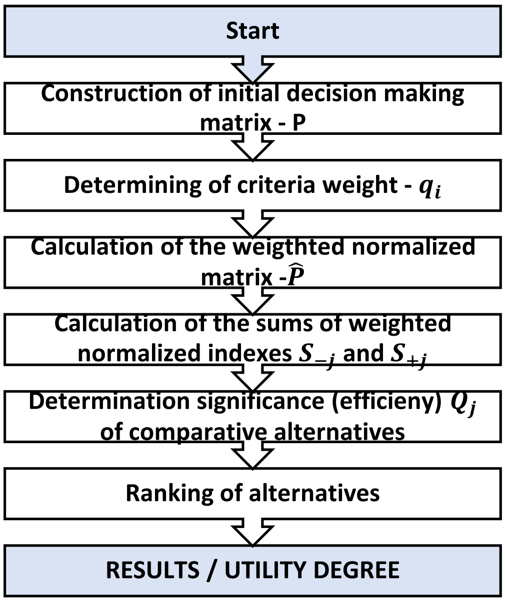

Figure 2.A schematic illustration of the COPRAS method steps[24]

KeshavarzGhorabaeeet al. proposed a multicriteria group decision-making approach using the interval type-2 fuzzy numbers based on the COPRAS method. Rani et al. proposed a sustainable supplier evaluation and selection approach based on COPRAS-SWARA[1] and the concept of a hesitant fuzzy set[25].The CRITIC[2] method is used to calculate the weight of the criteria. It extends the conventional COPRAS method to picture fuzzy systems to estimate each alternative's degree of desirability and select the optimal green supplier. Perçin examined the problem of choosinga circular supplier[26]. He proposes using IFSs to deal with the uncertainty that shapes decision-makers' judgment in solving the problem of selecting a circular supplier. He developed a group decision model based on the integrated AHP and COPRAS methodologies to consider three main criteria categories: economic, social, and circular. After using the AHP method to calculate the weight of individual criteria, he used the COPRAS method to rank potential suppliers. He tested the validity of the proposed method through a case study involving a multinational cement company. He conducted a sensitivity analysis to show the effect of the criteria weight on the presented approach results and compared the proposed method results with other MCDM methods. Ghosh et al. proposed a green supply chain management framework for evaluating supplier organizations[27]. They selected three Indian supplier organizations from three industrial sectors (service, manufacturing, and process organizations). They identified six criteria covering environmental, economic, and operational aspects of sustainability. They collected the data using a questionnaire, forming a committee of experts at the strategic, tactical, and operational levels. They used an integrated MCDM approach. The entropy method determines the criteria' weight, using a sophisticated proportional evaluation, a technique for prioritizing similarities to an ideal solution, and ranked supplier organizations by implementing grey relational analysis. The results show that "total energy consumption," "total scrap production," and "use of renewable energy" are effective parameters in choosing a green supplier. Jahangirzade et al. considered supplier selection in project procurement[28]. They proposed a new

[1]Step-wise Weight Assessment Ratio Analysis
[2] Criteria Importance Though Intercrieria Correlation

combination of COPRAS and GRA methods to select the best project supplier in uncertainty. They used a multi-objective optimization model to determine the weight of the criteria. They express the importance of decision-makers with a version of a combination of COPRAS and GRA methods. DEMİRCİ specified green suppliers' criteria in three sub-heading categories: resource utilization and green competence (green storage, green recycling, green production capacity, green packaging, resource consumption, pollution control), economic criteria (logistics costs, product costs, delivery time), and quality (error rate, warranty, and rights policies, environmental competencies, and documents). Then, the weighted criteria selected the most suitable green supplier, using COPRAS and EDAS1 methods. Ziquan et al. used intuitionistic fuzzy data to select shipbuilding enterprise suppliers[2].They used the intuitionistic fuzzy SWARA method to weigh each expert evaluation result according to their position, educational background, and working years. They then determined the ranking of suppliers by considering uncertainties and evaluating the utility index and the cost index of alternative suppliers using the intuitionistic fuzzy COPRAS method.

Ref.	Method	Objective
[1]	Ranking TI2FNs, based on the centroid method	Supplier selection
[3]	The intuitionistic fuzzy weighted averaging method	
[5]	Triangular Intuitionistic Fuzzy Decision-Making	Selecting a set of sustainability indicators
[7]	Fuzzy Goal Programming with TI2FNs	Solving multi-objective SSSPs
[10]	An intuitionistic fuzzy TOPSIS method	
[11]	Geographic Information System (GIS) and Multi-criteria Design Analysis (MCDA).	Identifying the flood hazard vulnerability zones
[12]	The study used AHPand Expert Choice software	Selecting the best alternative for Energy Technology
[13]	An integrated F-BWM and fuzzy CoCoSo with Bonferroni (CoCoSo'B)	SSP
[14]	Interval type-2 fuzzy AHP	Green supplier selection
[15]	A Borda Count-Based Intuitionistic Fuzzy EDAS-MAIRCA-MARCOS	Evaluation of Cryptocurrencies for Investment Decisions in the Era of Industry 4.0
[16]	Intuitionistic Fuzzy Sets and TO PSIS	Commercial-off-the Shelf Vendor Selection
[17]	MABACODAS method andTI2FNs	A risk quantification method in project-driven supply chain
[18]	MCDM methodology under intuitionistic fuzzy uncertainty	Selection of cloud service providers
[25]	Hesitant fuzzy SWARA	SSSP
[26]	Interval-valued intuitionistic fuzzy sets	Circular supplier selection
[27]	An integrated multivariate-MCDM approach	Evaluating the green suppliers
[28]	COPRAS and GRA methods	SSSP under a grey environment
[29]	A novel picture fuzzy COPRAS2	Selecting the green supplier
[30]	COPRAS and EDAS3	SSP
This paper	Using intuitionistic and fuzzy type 2 sets in COPRAS	Develop a group MCDM method for SSSP

[1] Evaluation based on Distance from Average Solution
[2]COmplexPRoportional Assessment
[3]The Evaluation Based on Distance from Average Solution

Past research did not consider the difference between the level of expertise of experts in different fields, while some experts may only have expertise in some fields, their opinion is not valid in other fields. This problem can cause the loss of validity of COPRAS model results. The innovation of this research is that it addresses the analysts' assessment of each expert's credibility and, in this regard, adds a complete perspective to the COPRAS model. It uses verbal variables to express the validity of the opinion of each of the decision-makers in each of the criteria. Also, another added value of this research is that by converting these verbal variables into intuitionistic fuzzy numbers, the uncertainty of experts' credibility is also shown. In addition to this, another contribution of this research is that it provides a suitable method for integrating experts' credibility in other COPRAS model information. The following sections explain the proposed model and compare the result of selecting sustainable suppliersby the proposed approach and the conventional fuzzy COPRAS method presented in KeshavarzGhorabaee et al. [1].

3. The proposed approach

Suppose C is a set of J criteria ($C = \{C_1, C_2, \dots, C_J\}$ and there are a set of n alternatives (suppliers)demonstrated by and k experts or decision-makers are a group that declares the alternatives scores in criteria and the criteria weights by verbal terms (linguistic variables). The model uses the following notations:

C: The set of j supplier selection criteria ($\mathrm{C} = \{\mathrm{C}_1, \mathrm{C}_2, \dots, \mathrm{C}_\mathrm{J}\}$)

A:The set of I potential suppliers (alternatives) ($A = \{\mathrm{A}_1, \mathrm{A}_2, \dots, \mathrm{A}_\mathrm{I}\}$)

D:The set of k decision-makers ($D = \{D_1, D_2, \dots, D_K\}$)

$\dot{w}_{jk}$: A linguistic variable explaining the weight of criteria C_j given by decision-maker D_k

$\dot{a}_{ijk}$:A linguistic variable describing the score of suppliersA_i in criteria C_j given by decision-maker D_k

$\dot{y}_{kj}$:A linguistic variable denoting the know-how of decision-maker D_k in criteria C_j given by the analyst

$\tilde{\tilde{y}}_{kj}$: An intuitionistic fuzzy number indicating the know-how of decision-maker D_k in criteria C_j

η_{kj}:The normalized values of $\tilde{\tilde{y}}_{kj}$

$\tilde{\tilde{x}}_{ij}$:The effective score of suppliersA_i in criteria C_j, aftersummarizing the opinions of decision-makers

$\tilde{\tilde{n}}_{ij}$: A trapezoidal interval type-2 fuzzy number indicating normalized values of $\tilde{\tilde{x}}_{ij}$

$\tilde{\tilde{e}}_{ij}$:The weighted normalized value of $\tilde{\tilde{x}}_{ij}$

$\tilde{\tilde{e}}_{+ij}$: The weighted normalized value of $\tilde{\tilde{x}}_{ij}$, when C_j is a beneficial criterion

$\tilde{\tilde{e}}_{-ij}$: The weighted normalized value of $\tilde{\tilde{x}}_{ij}$, when C_j is a non-beneficial criterion

$\tilde{\tilde{S}}_{+i}$: The total normalized weight of alternative i for beneficial criteria

$\tilde{\tilde{S}}_{-i}$: The total normalized weight of alternative *i* for non-beneficial criteria

Q_i:The relative significance of alternative A_i

Q_{max}: The maximum of the alternatives' significance

U_i: The relative importance of alternativeA_i

The analyst represents his evaluation of thevalidity ofexpert k's opinions on the criterion *j* by linguistic variables $\dot{y}_{kj}$. Table 1 illustrates the initial decision table of the COPRAS model, taking into account criteria weights and supplier scores given by the experts and the experts' credibility values provided bythe analyst.

Table 2. The COPRAS group decision-making matrix

	C_1			
	$\dot{W}_1 = \{\dot{w}_{11}, \dot{w}_{11}, \ldots \dot{w}_{1K}\}$			
	D_1	D_2	…	D_k
	$\dot{y}_{11}$	$\dot{y}_{21}$	…	$\dot{y}_{K1}$
A1	$\dot{a}_{111}$	$\dot{a}_{112}$	…	$\dot{a}_{11K}$
A2	$\dot{a}_{211}$	$\dot{a}_{212}$	…	$\dot{a}_{21K}$
…	…	…	…	…
AI	$\dot{a}_{I11}$	$\dot{a}_{I12}$	…	$\dot{a}_{I1K}$

	C_2			
	D_1	D_2	…	D_k
	$\dot{y}_{12}$	$\dot{y}_{22}$	…	$\dot{y}_{K2}$
A1	$\dot{a}_{121}$	$\dot{a}_{122}$	…	$\dot{a}_{12K}$
A2	$\dot{a}_{221}$	$\dot{a}_{222}$	…	$\dot{a}_{22K}$
…	…	…	…	…
AI	$\dot{a}_{I21}$	$\dot{a}_{I22}$	…	$\dot{a}_{I2K}$

	C_J			
	D_1	D_2	…	D_k
	$\dot{y}_{1J}$	$\dot{y}_{2J}$	…	$\dot{y}_{KJ}$
A1	$\dot{a}_{1J1}$	$\dot{a}_{1J2}$	…	$\dot{a}_{1JK}$
A2	$\dot{a}_{2J1}$	$\dot{a}_{2J2}$	…	$\dot{a}_{2JK}$
…	…	…	…	…
AI	$\dot{a}_{IJ1}$	$\dot{a}_{IJ2}$	…	$\dot{a}_{IJK}$

Table 2 shows the equivalent of linguistic variables as type 2 fuzzy variables.

Table 3. Conversion of linguistic variables assessing the importance of criteria to interval type-2 fuzzy sets

	Interval type-2 fuzzy number ($\tilde{\tilde{a}}_{ijk}$ **or** $\tilde{\tilde{w}}_j^k$)
Very low(VL)	((0,0,0,0.1;1,1),(0,0,0,0.05;0.9,0.9))
Low(L)	((0,0.1,0.15,0.3;1,1),(0.05,0.1,0.15,0.2;0.9,0.9))
Medium-low (ML)	((0.1,0.3,0.35,0.5;1,1),(0.2,0.3,0.35,0.4;0.9,0.9))
Medium (M)	((0.3,0.5,0.55,0.7;1,1),(0.4,0.5,0.55,0.6;0.9,0.9))
Medium-high (MH)	((0.5,0.7,0.75,0.9;1,1)),(0.6,0.7,0.75,0.8;0.9,0.9))
High (H)	((0.7,0.85,0.9,1;1,1)),(0.8,0.85,0.9,0.95;0.9,0.9))
Very high (VH)	((0.9,1,1,1;1,1),(0.95,1,1,1;0.9,0.9))

The analyst also expresses his opinion about the validity of each expert with a verbal variable($\dot{y}_{kj}$) that in later stages becomes intuitionistic fuzzy number$\tilde{\tilde{y}}_{kj} = (\mu_{kj}, \nu_{kj})$using Table 3.

Using intuitionistic fuzzy numbers and expressing the degree of membership of possible values also indicates the analyst's lack of knowledge or hesitation in estimating the credibility of experts as non-membership values. The values of$\mu_{kj}$and ν_{kj}, respectively, indicate the degree of membership and non-membership of expert k in the set of professionals in the field of j.

Table 4. The equivalent intuitionistic fuzzy sets for the linguistic variables

Linguistic variables ($\dot{y}_{kj}$)	**Intuitionistic fuzzy set** ($\tilde{\tilde{y}}_{kj}$)
Very Important (*VI*)	(0.90, 0.10)
Important (*I*)	(0.75, 0.20)
Medium (*M*)	(0.50, 0.45)
Unimportant (*U*)	(0.35, 0.60)
VeryUnimportant(*VU*)	(0.10, 0.90)

Step 1. Convert the verbal variables representing the validity of experts to IFNs, using Table 3, and defuzzify then the resulting IFNs by the following formula:

$$d(\tilde{\tilde{y}}_{kj}) = \left(\mu_{kj} + \pi_{kj}\left(\frac{\mu_{kj}}{\mu_{kj+\nu_{kj}}}\right)\right) \quad j=1, 2, \ldots J \quad k=1,2,..,K \tag{5}$$

Step 2. Normalize the credibility of each expert by using the following equation:

$$\eta_{kj} = \frac{d(\tilde{\tilde{y}}_{kj})}{\sum_{k'=1}^{K} d(\tilde{\tilde{y}}_{k'j})} \quad j=1, 2, \ldots, J \quad k=1, 2, ..., K \tag{6}$$

Where $\sum_{k=1}^{K} \eta_{kj} = 1$ for j=1,2,…,J. Then the effective score of each supplier in each criterion is calculated by each expert from the following equation:

$$\tilde{\tilde{x}}_{ij} = \sum_{k=1}^{K} \eta_{kj}\tilde{\tilde{a}}_{ijk} \quad i=1,2,\ldots,I \quad j=1, 2, \ldots, J \tag{7}$$

Step 3. Display the mean decision matrix as follows:

$$\tilde{\tilde{X}} = [\tilde{\tilde{x}}_{ij}]_{I\times J} \tag{8}$$

Step 4. Normalized decision matrix calculation $\tilde{\tilde{N}}$ by using the following fuzzy[1]:

$$\tilde{\tilde{x}}_j = \tilde{\tilde{x}}_{1j} \oplus \tilde{\tilde{x}}_{2j} \oplus \ldots \oplus \tilde{\tilde{x}}_{Ij} \quad j=1,2,\ldots,J \tag{9}$$

$$\tilde{\tilde{n}}_{ij} = \frac{\tilde{\tilde{x}}_{ij}}{\tilde{\tilde{x}}_j} \quad i=1,2,\ldots,I, \ j=1,2,..J \tag{10}$$

$$N = [n_{ij}]_{I\times J} \tag{11}$$

Step 5. Display the verbal criteria weight matrix obtained from the decision-makers as follows:

$$\dot{W}^k = [\dot{w}_j^k]_{J\times 1} \quad k=1, 2, ..., K \tag{12}$$

By using Table 2, the analyst transfersthe verbal weights to the following TI2FN weight matrix:

$$\tilde{\tilde{W}}^k = [\tilde{\tilde{w}}_j^k]_{J\times 1} \quad k=1, 2, ..., K \tag{13}$$

Step 6. Calculate the following average weight matrix:

$$\tilde{\tilde{w}}_j = (\tilde{\tilde{w}}_j^1 \oplus \tilde{\tilde{w}}_j^1 \oplus \ldots \oplus \tilde{\tilde{w}}_j^K)/K \quad j=1,2,..J \tag{14}$$

$$\tilde{\tilde{W}} = [\tilde{\tilde{w}}_j]_{j\times j} \tag{15}$$

Step 7. Calculate the weighted normalized decision matrix elements as follows:

$$\tilde{\tilde{e}}_{ij} = \tilde{\tilde{n}}_{ij} \otimes \tilde{\tilde{w}}_{ij} \quad i=1,2,\ldots,I, j=1,2,..J \tag{16}$$

$$\tilde{\tilde{E}} = [\tilde{\tilde{e}}_{ij}]_{I\times J} \tag{17}$$

Step 8. The values of the total normalized weight of alternatives for beneficial and non-beneficial criteria are as follows:

$$\tilde{\tilde{S}}_{+i} = (\tilde{\tilde{e}}_{+i1} \oplus \tilde{\tilde{e}}_{+i2} \oplus \ldots . \oplus \tilde{\tilde{e}}_{+im}) \quad i=1,2,\ldots,I \tag{18}$$

$$\tilde{\tilde{S}}_{-i} = (\tilde{\tilde{e}}_{-i1} \oplus \tilde{\tilde{e}}_{-i2} \oplus \ldots . \oplus \tilde{\tilde{e}}_{-im}) \quad i=1,2,\ldots,I \tag{19}$$

Where $\tilde{\tilde{e}}_{-ij}$and $\tilde{\tilde{e}}_{+ij}$ are the weighted normalized values for the beneficial and non-beneficial, and smaller$\tilde{\tilde{S}}_{-i}$, the better scores for alternative i.

Step 9. Calculate the ranking values for both $\tilde{\tilde{S}}_{+i}$ and$\tilde{\tilde{S}}_{-i}$as follows:

$$Rank(A_i) = \frac{1}{I(I-1)}\left(\sum_{i=1}^{I} P(A_i > A_j)\right) + \frac{I}{2} - 1 \quad i=1,2,\ldots,I \tag{20}$$

Where $P(A_i \geq A_{i'})$ is the possibility of $A_i \geq A_{i'}$, explained in KeshavarzGhorabaeeet al. [1].

Step 10.The relative significance of each alternative (Q_i, i=1, 2, …, I) is determined using the following equation[1]:

$$Q_i = Rank(S_{+i}) + \frac{\sum_{i=1}^{n}(Rank(S_{-i}))}{Rank(S_{-i})\sum_{i=1}^{n}(\frac{1}{Rank(S_{-i})})} \tag{21}$$

Step 11. The degree of satisfaction is attained by the alternatives determined, using the following equation:

$$U_i = \left(\frac{Q_i}{Q_{\max}}\right) \times 100 \quad i=1,2,\ldots,I \tag{22}$$

In the above equation, $Q_{max}$is the maximum of the alternatives' importance andU_i is the alternatives' relative importance. The larger U_i, the more preferred alternative i is.

4. Numerical examples

A manufacturing company wants to select the most appropriate supplier using sustainability criteria to purchase the main components of its new products. After the initial screening, three candidates (A_1, A_2, A_3) remain for further evaluation. The analyst uses the verbal scores and the verbal criteria weights ($\dot{a}_{ijk}$ and $\dot{w}_j^k$) given by the three decision-makers D_1, D_2 andD_3 in the context of three economic, social, and environmental criteria to select one of the three proposed suppliers A_1, A_2 andA_3.Please note that "social" and "environmental" criteria are desirable, and "economic" criteria are undesirable. Each TI2FN in the right column of the above table is in the following format:

$$\begin{aligned}\tilde{\tilde{w}}_j &= (\tilde{\tilde{w}}_j^U, \tilde{\tilde{w}}_j^L) \\ &= ((w_{1\,j}^U, w_{2\,j}^U, w_{3\,j}^U, w_{4\,j}^U; H_{1\,j}(\tilde{w}_j^U), H_2(\tilde{w}_j^U)), \\ &\quad ((w_{1\,j}^L, w_{2\,j}^L, w_{3\,j}^L, w_{4\,j}^L; H_{1\,j}(\tilde{w}_j^L), H_2(\tilde{w}_j^L)))\end{aligned} \tag{23}$$

Table 4 shows the validity of each expert's opinion from the analyst's point of view in each criterion in the form of verbal variables.

Table 5. Validity of each expert in each criterion

	D_1	D_2	D_3
C1	I	VI	VU
C2	VI	M	M
C3	U	VU	M

First, using equation (6), obtain the validity of each expert and then normalize it, using Equation 7; Table 5 shows the results.

Table 6. Normalized validity of each expert in each criterion

			D_1	D_2	D_3
Expert credit	**Econo. Criterion**	**Original**	0.79	0.1	0.90
		Normalized	0.44	0.06	0.50
	Social Criterion	**Original**	0.90	0.53	0.53
		Normalized	0.46	0.27	0.27
	Environ. Criterion	**Original**	0.37	0.10	0.53
		Normalized	0.37	0.1	0.53

Table 6 shows the weights of the criteria given by the experts as linguistic variables.

Table 7. Weight of the criteria

	Experts		
Criterion	D_1	D_2	D_3
Economical	VH	H	VH
Social	MH	M	M
Environmental	H	H	MH

Now using Equations 14 and 15, the mean of the decision matrix$\tilde{\tilde{\bar{W}}}$is as follows:

Table 8. Mean weight matrix $\left(\tilde{\tilde{\bar{W}}}\right)$

	$\tilde{W}_{kij}^U$					
	W_{1ij}^U	W_{2ij}^U	W_{3ij}^U	W_{4ij}^U	$H_1(\tilde{W}_{ij}^U)$	$H_2(\tilde{W}_{ij}^U)$
$\tilde{\tilde{W}}_1$	0.83	0.95	0.97	1.00	1.00	1.00
$\tilde{\tilde{W}}_2$	0.37	0.57	0.62	0.77	1.00	1.00
$\tilde{\tilde{W}}_3$	0.63	0.80	0.85	0.97	1.00	1.00

	$\tilde{W}_{kij}^L$					
	W_{1ij}^L	W_{2ij}^L	W_{3ij}^L	W_{4ij}^L	$H_1(\tilde{W}_{ij}^L)$	$H_2(\tilde{W}_{ij}^L)$
$\tilde{\tilde{W}}_1$	0.90	0.95	0.97	0.98	0.90	0.90
$\tilde{\tilde{W}}_2$	0.47	0.57	0.62	0.67	0.90	0.90
$\tilde{\tilde{W}}_3$	0.73	0.80	0.85	0.90	0.90	0.90

Table 8 shows the expert opinion on suppliers' economic, social, and environmental criteria scores.

Table 9. The suppliers' scores given by experts in three decision criteria

	Economical			Social			Environmental		
	D_1	D_2	D_3	D_1	D_2	D_3	D_1	D_2	D_3
D_{kj}	0.44	0.06	0.50	0.46	0.27	0.27	0.37	0.10	0.53
A_1	H	ML	MH	L	M	ML	M	ML	M
A_2	MH	H	VH	ML	ML	VL	L	L	VL
A_3	MH	M	M	M	M	ML	ML	L	L
A4	H	H	MH	L	ML	VL	M	L	M
A5	MH	M	VH	ML	M	ML	L	L	VL

For example, the number 0.44 in Table 8 is calculated from the following equation:

$$0.44 = \frac{0.79}{0.79 + 0.1 + 0.9}$$

Table 9 shows the result of calculating the mean decision matrix based on the table above and Equation 8.

Table 10. Mean decision matrix ($\tilde{\tilde{X}}$)

	$\tilde{x}^U_{kij}$					
	x^U_{1ij}	x^U_{2ij}	x^U_{3ij}	x^U_{4ij}	$H_1(\tilde{x}^U_{ij})$	$x_2(\tilde{x}^U_{ij})$
$\tilde{\tilde{x}}_{11}$	0.19	0.25	0.26	0.31	1.00	1.00
$\tilde{\tilde{x}}_{12}$	0.24	0.29	0.29	0.32	1.00	1.00
$\tilde{\tilde{x}}_{13}$	0.13	0.20	0.21	0.26	1.00	1.00
$\tilde{\tilde{x}}_{14}$	0.20	0.26	0.27	0.32	1.00	1.00
$\tilde{\tilde{x}}_{15}$	0.23	0.28	0.29	0.31	1.00	1.00
$\tilde{\tilde{x}}_{21}$	0.04	0.09	0.10	0.15	1.00	1.00
$\tilde{\tilde{x}}_{22}$	0.02	0.07	0.09	0.13	1.00	1.00
$\tilde{\tilde{x}}_{23}$	0.08	0.15	0.17	0.22	1.00	1.00
$\tilde{\tilde{x}}_{24}$	0.01	0.04	0.05	0.10	1.00	1.00
$\tilde{\tilde{x}}_{25}$	0.05	0.12	0.13	0.18	1.00	1.00
$\tilde{\tilde{x}}_{31}$	0.09	0.16	0.18	0.23	1.00	1.00
$\tilde{\tilde{x}}_{32}$	0.00	0.02	0.02	0.06	1.00	1.00
$\tilde{\tilde{x}}_{33}$	0.01	0.06	0.07	0.12	1.00	1.00
$\tilde{\tilde{x}}_{34}$	0.01	0.04	0.05	0.10	1.00	1.00
$\tilde{\tilde{x}}_{35}$	0.05	0.12	0.13	0.18	1.00	1.00
	$\tilde{x}^L_{kij}$					
	x^L_{1ij}	x^L_{2ij}	x^L_{3ij}	x^L_{4ij}	$H_1(\tilde{x}^L_{ij})$	$H_2(\tilde{x}^L_{ij})$
$\tilde{\tilde{x}}_{11}$	0.22	0.25	0.26	0.28	0.90	0.90
$\tilde{\tilde{x}}_{12}$	0.26	0.29	0.29	0.30	0.90	0.90
$\tilde{\tilde{x}}_{13}$	0.16	0.20	0.21	0.23	0.90	0.90
$\tilde{\tilde{x}}_{14}$	0.23	0.26	0.27	0.29	0.90	0.90
$\tilde{\tilde{x}}_{15}$	0.25	0.28	0.29	0.30	0.90	0.90
$\tilde{\tilde{x}}_{21}$	0.06	0.09	0.10	0.12	0.90	0.90
$\tilde{\tilde{x}}_{22}$	0.05	0.07	0.09	0.10	0.90	0.90
$\tilde{\tilde{x}}_{23}$	0.12	0.15	0.17	0.18	0.90	0.90
$\tilde{\tilde{x}}_{24}$	0.03	0.04	0.05	0.07	0.90	0.90
$\tilde{\tilde{x}}_{25}$	0.08	0.12	0.13	0.15	0.90	0.90
$\tilde{\tilde{x}}_{31}$	0.13	0.16	0.18	0.19	0.90	0.90
$\tilde{\tilde{x}}_{32}$	0.01	0.02	0.02	0.04	0.90	0.90
$\tilde{\tilde{x}}_{33}$	0.04	0.06	0.07	0.09	0.90	0.90
$\tilde{\tilde{x}}_{34}$	0.03	0.04	0.05	0.07	0.90	0.90
$\tilde{\tilde{x}}_{35}$	0.08	0.12	0.13	0.15	0.90	0.90

For example, the number 0.19in Table 9 is calculated from the following equation:

$$\frac{H + ML + MH}{3} = 0.19$$

Table 10 shows the normalized decision matrix using Equations9,10,11:

Table 11. The normalized decision matrix ($\tilde{\tilde{N}}$)

	$\tilde{x}^U_{kij}$					
	n^U_{1ij}	n^U_{2ij}	n^U_{3ij}	n^U_{4ij}	$H_1(\tilde{n}^U_{ij})$	$n_2(\tilde{n}^U_{ij})$
$\tilde{\tilde{n}}_{11}$	0.12	0.19	0.21	0.31	1.00	1.00
$\tilde{\tilde{n}}_{12}$	0.16	0.21	0.23	0.32	1.00	1.00
$\tilde{\tilde{n}}_{13}$	0.09	0.15	0.17	0.27	1.00	1.00
$\tilde{\tilde{n}}_{14}$	0.13	0.19	0.22	0.32	1.00	1.00
$\tilde{\tilde{n}}_{15}$	0.15	0.21	0.23	0.32	1.00	1.00
$\tilde{\tilde{n}}_{21}$	0.05	0.16	0.22	0.76	1.00	1.00
$\tilde{\tilde{n}}_{22}$	0.03	0.13	0.18	0.64	1.00	1.00
$\tilde{\tilde{n}}_{23}$	0.10	0.27	0.35	1.06	1.00	1.00
$\tilde{\tilde{n}}_{24}$	0.01	0.08	0.12	0.49	1.00	1.00
$\tilde{\tilde{n}}_{25}$	0.07	0.22	0.29	0.91	1.00	1.00
$\tilde{\tilde{n}}_{31}$	0.13	0.34	0.44	1.16	1.00	1.00
$\tilde{\tilde{n}}_{32}$	0.00	0.03	0.06	0.33	1.00	1.00
$\tilde{\tilde{n}}_{33}$	0.02	0.12	0.19	0.64	1.00	1.00
$\tilde{\tilde{n}}_{34}$	0.13	0.33	0.42	1.12	1.00	1.00
$\tilde{\tilde{n}}_{35}$	0.00	0.03	0.06	0.33	1.00	1.00
	$\tilde{x}^L_{kij}$					
	n^L_{1ij}	n^L_{2ij}	n^L_{3ij}	n^L_{4ij}	$H_1(\tilde{n}^L_{ij})$	$H_2(\tilde{n}^L_{ij})$
$\tilde{\tilde{n}}_{11}$	0.16	0.19	0.21	0.25	0.90	0.90
$\tilde{\tilde{n}}_{12}$	0.19	0.21	0.23	0.27	0.90	0.90
$\tilde{\tilde{n}}_{13}$	0.12	0.15	0.17	0.20	0.90	0.90
$\tilde{\tilde{n}}_{14}$	0.17	0.19	0.22	0.26	0.90	0.90
$\tilde{\tilde{n}}_{15}$	0.18	0.21	0.23	0.26	0.90	0.90
$\tilde{\tilde{n}}_{21}$	0.10	0.16	0.22	0.36	0.90	0.90
$\tilde{\tilde{n}}_{22}$	0.08	0.13	0.18	0.30	0.90	0.90
$\tilde{\tilde{n}}_{23}$	0.18	0.27	0.35	0.54	0.90	0.90
$\tilde{\tilde{n}}_{24}$	0.04	0.08	0.12	0.21	0.90	0.90
$\tilde{\tilde{n}}_{25}$	0.14	0.22	0.29	0.45	0.90	0.90
$\tilde{\tilde{n}}_{31}$	0.23	0.34	0.44	0.65	0.90	0.90
$\tilde{\tilde{n}}_{32}$	0.01	0.03	0.06	0.13	0.90	0.90
$\tilde{\tilde{n}}_{33}$	0.06	0.12	0.19	0.31	0.90	0.90
$\tilde{\tilde{n}}_{34}$	0.22	0.33	0.42	0.62	0.90	0.90
$\tilde{\tilde{n}}_{35}$	0.01	0.03	0.06	0.13	0.90	0.90

For example, the number 0.12 in Table 10 is calculated from $X_{11} \oslash (\tilde{\tilde{X}}_{11} \oplus \tilde{\tilde{X}}_{12} \oplus \tilde{\tilde{X}}_{13} \oplus \tilde{\tilde{X}}_{14} \oplus \tilde{\tilde{X}}_{15}) =$

0.12formula. By using equations 16 and 17 the weight normalization decision matrix ($\tilde{\tilde{E}}$) is as follows:

Table 12. Weighted normalized decision matrix ($\tilde{\tilde{E}}$)

	$\tilde{n}^U_{kij}$					
	e^U_{1ij}	e^U_{2ij}	e^U_{3ij}	e^U_{4ij}	$H_1(\tilde{e}^U_{ij})$	$H_2(\tilde{e}^U_{ij})$
$\tilde{\tilde{e}}_{11}$	0.10	0.18	0.31	0.31	1.00	1.00
$\tilde{\tilde{e}}_{12}$	0.13	0.20	0.32	0.32	1.00	1.00
$\tilde{\tilde{e}}_{13}$	0.07	0.14	0.27	0.27	1.00	1.00
$\tilde{\tilde{e}}_{14}$	0.11	0.18	0.32	0.32	1.00	1.00
$\tilde{\tilde{e}}_{15}$	0.13	0.20	0.32	0.32	1.00	1.00
$\tilde{\tilde{e}}_{21}$	0.02	0.09	0.58	0.58	1.00	1.00
$\tilde{\tilde{e}}_{22}$	0.01	0.08	0.49	0.49	1.00	1.00
$\tilde{\tilde{e}}_{23}$	0.04	0.15	0.81	0.81	1.00	1.00
$\tilde{\tilde{e}}_{24}$	0.00	0.04	0.38	0.38	1.00	1.00
$\tilde{\tilde{e}}_{25}$	0.02	0.12	0.70	0.70	1.00	1.00
$\tilde{\tilde{e}}_{31}$	0.08	0.27	1.12	1.12	1.00	1.00
$\tilde{\tilde{e}}_{32}$	0.00	0.03	0.32	0.32	1.00	1.00
$\tilde{\tilde{e}}_{33}$	0.01	0.10	0.62	0.62	1.00	1.00
$\tilde{\tilde{e}}_{34}$	0.08	0.26	1.09	1.09	1.00	1.00
$\tilde{\tilde{e}}_{35}$	0.00	0.03	0.32	0.32	1.00	1.00
	$\tilde{n}^L_{kij}$					
	e^L_{1ij}	e^L_{2ij}	e^L_{3ij}	e^L_{4ij}	$H_1(\tilde{e}^L_{ij})$	$H_2(\tilde{e}^L_{ij})$
$\tilde{\tilde{e}}_{11}$	0.14	0.18	0.24	0.24	0.90	0.90
$\tilde{\tilde{e}}_{12}$	0.17	0.20	0.26	0.26	0.90	0.90
$\tilde{\tilde{e}}_{13}$	0.10	0.14	0.20	0.20	0.90	0.90
$\tilde{\tilde{e}}_{14}$	0.15	0.18	0.25	0.25	0.90	0.90
$\tilde{\tilde{e}}_{15}$	0.16	0.20	0.26	0.26	0.90	0.90
$\tilde{\tilde{e}}_{21}$	0.05	0.09	0.24	0.24	0.90	0.90
$\tilde{\tilde{e}}_{22}$	0.04	0.08	0.20	0.20	0.90	0.90
$\tilde{\tilde{e}}_{23}$	0.09	0.15	0.36	0.36	0.90	0.90
$\tilde{\tilde{e}}_{24}$	0.02	0.04	0.14	0.14	0.90	0.90
$\tilde{\tilde{e}}_{25}$	0.06	0.12	0.30	0.30	0.90	0.90
$\tilde{\tilde{e}}_{31}$	0.17	0.27	0.58	0.58	0.90	0.90
$\tilde{\tilde{e}}_{32}$	0.01	0.03	0.12	0.12	0.90	0.90
$\tilde{\tilde{e}}_{33}$	0.05	0.10	0.27	0.27	0.90	0.90
$\tilde{\tilde{e}}_{34}$	0.16	0.26	0.56	0.56	0.90	0.90
$\tilde{\tilde{e}}_{35}$	0.01	0.03	0.12	0.12	0.90	0.90

Forexample, the number 0.10 in Table 11 is calculated from equation0.10= $\tilde{n}_{11} \otimes \tilde{\tilde{W}}_1$. The values of $\tilde{\tilde{S}}_{+i}$ and $\tilde{\tilde{S}}_{-i}$ are calculated based on equations 18 and 19, and the results are shown in Tables 12 and 13

Table 1. The $\tilde{\tilde{S}}_{+i}$ value

	$\tilde{S}^U_{+i}$					
	S^U_{1+i}	S^U_{2+i}	S^U_{3+i}	S^U_{4+i}	$H_1(\tilde{S}^U_{+i})$	$H_2(\tilde{S}^U_{+i})$
$\tilde{\tilde{S}}_{+1}$	0.10	0.36	1.70	1.70	1.00	1.00
$\tilde{\tilde{S}}_{+2}$	0.01	0.10	0.81	0.81	1.00	1.00
$\tilde{\tilde{S}}_{+3}$	0.05	0.25	1.43	1.43	1.00	1.00
$\tilde{\tilde{S}}_{+4}$	0.09	0.31	1.47	1.47	1.00	1.00
$\tilde{\tilde{S}}_{+5}$	0.02	0.15	1.02	1.02	1.00	1.00
	$\tilde{S}^L_{+i}$					
	S^L_{1+i}	S^L_{2+i}	S^L_{3+i}	S^L_{4+i}	$H_1(\tilde{S}^L_{+i})$	$H_2(\tilde{S}^L_{+i})$
$\tilde{\tilde{S}}_{+1}$	0.21	0.36	0.82	0.82	0.90	0.90
$\tilde{\tilde{S}}_{+2}$	0.05	0.10	0.32	0.32	0.90	0.90
$\tilde{\tilde{S}}_{+3}$	0.13	0.25	0.64	0.64	0.90	0.90
$\tilde{\tilde{S}}_{+4}$	0.18	0.31	0.70	0.70	0.90	0.90
$\tilde{\tilde{S}}_{+5}$	0.07	0.15	0.42	0.42	0.90	0.90

For example, the number 0.10 in Table 12 is calculated from equation0.10= $\tilde{\tilde{e}}_{21} \oplus \tilde{\tilde{e}}_{31}$.

Table 2. The $\tilde{\tilde{S}}_{-i}$value

	$\tilde{S}^U_{-i}$					
	S^U_{1-i}	S^U_{2-i}	S^U_{3-i}	S^U_{4-i}	$H_1(\tilde{S}^U_{-i})$	$H_2(\tilde{S}^U_{-i})$
$\tilde{\tilde{S}}_{-1}$	0.10	0.18	0.31	0.31	1.00	1.00
$\tilde{\tilde{S}}_{-2}$	0.13	0.20	0.32	0.32	1.00	1.00
$\tilde{\tilde{S}}_{-3}$	0.07	0.14	0.27	0.27	1.00	1.00
$\tilde{\tilde{S}}_{-4}$	0.11	0.18	0.32	0.32	1.00	1.00
$\tilde{\tilde{S}}_{-5}$	0.13	0.20	0.32	0.32	1.00	1.00
	$\tilde{S}^L_{-i}$					
	S^U_{1-i}	S^U_{2-i}	S^U_{3-i}	S^U_{4-i}	$H_1(\tilde{S}^U_{-i})$	$H_2(\tilde{S}^U_{-i})$
$\tilde{\tilde{S}}_{-1}$	0.10	0.18	0.31	0.31	1.00	1.00
$\tilde{\tilde{S}}_{-2}$	0.13	0.20	0.32	0.32	1.00	1.00
$\tilde{\tilde{S}}_{-3}$	0.07	0.14	0.27	0.27	1.00	1.00
$\tilde{\tilde{S}}_{-4}$	0.11	0.18	0.32	0.32	1.00	1.00
$\tilde{\tilde{S}}_{-5}$	0.13	0.20	0.32	0.32	1.00	1.00

For example, the number 0.10 in Table 13 is calculated from equation0.10= $\tilde{\tilde{e}}_{11}$.Using equation 20, Table 14 shows the ranking of $\boldsymbol{Rank}\left(\tilde{\tilde{S}}_{+i}\right)$and $\boldsymbol{Rank}\left(\tilde{\tilde{S}}_{-i}\right)$, calculated.

Table 14. The ranking $\tilde{\tilde{S}}_{+i}$ and $\tilde{\tilde{S}}_{-i}$values

i	$Rank\left(\tilde{\tilde{S}}_{+i}\right)$	$Rank\left(\tilde{\tilde{S}}_{-i}\right)$
1	0.32	0.32
2	0.30	0.32
3	0.32	0.31
4	0.32	0.32
5	0.31	0.32

For example, the number 0.32 in Table 14 is calculated from the following equation:

$$Rank\left(\tilde{\tilde{S}}_{+i}\right) = 0.32 = \frac{1}{20} = (1 + 1 + 1 + 1 + 1 + 1.5)$$

and

$$Rank\left(\tilde{\tilde{S}}_{-i}\right) = 0.32 = \frac{1}{20}(1 + 0.98 + 1 + 0.99 + 0.98) + 1.5)$$

Table 15 illustrates, the $Q_i$and$U_i$values calculated using equations 21 and 22.For example, the number 0.65 and 0.99 in Table 15 is calculated from the following equation:

$$0.65=0.32+\frac{(0.32+0.32+0.31)}{0.32\left(\frac{1}{0.32}+\frac{1}{0.32}+\frac{1}{0.31}\right)}\text{and } 0.99=\frac{0.65}{0.65}\times 100.$$

Table 15. The $Q_i$and$U_i$values

i	Q_i	U_i
1	0.65	99
2	0.61	94
3	0.65	100
4	0.64	98
5	0.63	96

Therefore, according to Table 15, the optimal final ranking of the options is as follows:

$$A_3 > A_1 > A_4 > A_5 > A_2 \quad (24)$$

The above example is resolved with the original model presented byKeshavarzGhorabaeeet al. [1]. The difference between the original method and the method proposed in this paper is that the decision-makers have the same validity, and the analyst's point of view has no on the decision-making results.The original method does not require steps 1 and 2 of the proposed method, and $\tilde{\tilde{X}}_{ij}^{k}$ is the value of the supplier $A_i$score in the criterion $C_j$given by decision-maker D_k, and the decision matrix is as follows:

Table 16. The scores of suppliers ($\tilde{\tilde{X}}_{ij}^{k}$)given by experts

	Economical			Social			Environmental		
	D_1	D_2	D_3	D_1	D_2	D_3	D_1	D_2	D_3
A_1	H	ML	MH	L	M	ML	M	ML	M
A_2	MH	H	VH	ML	ML	VL	L	L	VL
A_3	MH	M	M	M	M	ML	ML	L	L
A_4	H	H	MH	L	ML	VL	M	L	M
A_5	MH	M	VH	ML	M	ML	L	L	VL

Table 17 shows the result of calculating the summarized Mean decision matrix based on the information in the above table by using Equation 8.

Table 17. The Mean decision matrix ($\bar{\tilde{\tilde{X}}}$)

	x_{1ij}^{U}	x_{2ij}^{U}	x_{3ij}^{U}	x_{4ij}^{U}	$H_1(\tilde{x}_{ij}^{U})$	$H_2(\tilde{x}_{ij}^{U})$
$\tilde{\tilde{x}}_{11}$	0.14	0.21	0.22	0.27	1.00	1.00
$\tilde{\tilde{x}}_{12}$	0.23	0.28	0.29	0.32	1.00	1.00
$\tilde{\tilde{x}}_{13}$	0.12	0.19	0.21	0.26	1.00	1.00
$\tilde{\tilde{x}}_{14}$	0.21	0.27	0.28	0.32	1.00	1.00
$\tilde{\tilde{x}}_{15}$	0.19	0.24	0.26	0.29	1.00	1.00
$\tilde{\tilde{x}}_{21}$	0.04	0.10	0.12	0.17	1.00	1.00
$\tilde{\tilde{x}}_{22}$	0.02	0.07	0.08	0.12	1.00	1.00
$\tilde{\tilde{x}}_{23}$	0.08	0.14	0.16	0.21	1.00	1.00
$\tilde{\tilde{x}}_{24}$	0.01	0.04	0.06	0.10	1.00	1.00
$\tilde{\tilde{x}}_{25}$	0.06	0.12	0.14	0.19	1.00	1.00
$\tilde{\tilde{x}}_{31}$	0.08	0.14	0.16	0.21	1.00	1.00
$\tilde{\tilde{x}}_{32}$	0.00	0.02	0.03	0.08	1.00	1.00
$\tilde{\tilde{x}}_{33}$	0.01	0.06	0.07	0.12	1.00	1.00
$\tilde{\tilde{x}}_{34}$	0.07	0.12	0.14	0.19	1.00	1.00
$\tilde{\tilde{x}}_{35}$	0.00	0.02	0.03	0.08	1.00	1.00
			$\tilde{x}_{kij}^{L}$			
	x_{1ij}^{L}	x_{2ij}^{L}	x_{3ij}^{L}	x_{4ij}^{L}	$H_1(\tilde{x}_{ij}^{L})$	$H_2(\tilde{x}_{ij}^{L})$
$\tilde{\tilde{x}}_{11}$	0.18	0.21	0.22	0.24	0.90	0.90
$\tilde{\tilde{x}}_{12}$	0.26	0.28	0.29	0.31	0.90	0.90
$\tilde{\tilde{x}}_{13}$	0.16	0.19	0.21	0.22	0.90	0.90
$\tilde{\tilde{x}}_{14}$	0.24	0.27	0.28	0.30	0.90	0.90
$\tilde{\tilde{x}}_{15}$	0.22	0.24	0.26	0.27	0.90	0.90
$\tilde{\tilde{x}}_{21}$	0.07	0.10	0.12	0.13	0.90	0.90
$\tilde{\tilde{x}}_{22}$	0.04	0.07	0.08	0.09	0.90	0.90
$\tilde{\tilde{x}}_{23}$	0.11	0.14	0.16	0.18	0.90	0.90
$\tilde{\tilde{x}}_{24}$	0.03	0.04	0.06	0.07	0.90	0.90
$\tilde{\tilde{x}}_{25}$	0.09	0.12	0.14	0.16	0.90	0.90
$\tilde{\tilde{x}}_{31}$	0.11	0.14	0.16	0.18	0.90	0.90
$\tilde{\tilde{x}}_{32}$	0.01	0.02	0.03	0.05	0.90	0.90
$\tilde{\tilde{x}}_{33}$	0.03	0.06	0.07	0.09	0.90	0.90
$\tilde{\tilde{x}}_{34}$	0.09	0.12	0.14	0.16	0.90	0.90
$\tilde{\tilde{x}}_{35}$	0.01	0.02	0.03	0.05	0.90	0.90

For example, the number 0.14 in Table 17 is calculated from the following equation:

$$0.14=\frac{H+ML+MH}{3}$$

According to the results of the previous example, the average weight of the criteria remains the same as in Table 7, and Table 18 presents the normalized decision matrix.

Table 18. Normalized decision matrix ($\tilde{\tilde{N}}$)

	$\tilde{n}^U_{kij}$					
	n^U_{1ij}	n^U_{2ij}	n^U_{3ij}	n^U_{4ij}	$H_1(\tilde{n}^U_{ij})$	$H_2(\tilde{n}^U_{ij})$
$\tilde{\tilde{n}}_{11}$	0.10	0.16	0.19	0.30	1.00	1.00
$\tilde{\tilde{n}}_{12}$	0.16	0.22	0.25	0.36	1.00	1.00
$\tilde{\tilde{n}}_{13}$	0.08	0.15	0.17	0.28	1.00	1.00
$\tilde{\tilde{n}}_{14}$	0.15	0.21	0.24	0.36	1.00	1.00
$\tilde{\tilde{n}}_{15}$	0.13	0.19	0.21	0.32	1.00	1.00
$\tilde{\tilde{n}}_{21}$	0.06	0.18	0.24	0.79	1.00	1.00
$\tilde{\tilde{n}}_{22}$	0.03	0.12	0.16	0.58	1.00	1.00
$\tilde{\tilde{n}}_{23}$	0.10	0.26	0.34	1.00	1.00	1.00
$\tilde{\tilde{n}}_{24}$	0.01	0.08	0.12	0.47	1.00	1.00
$\tilde{\tilde{n}}_{25}$	0.07	0.22	0.29	0.89	1.00	1.00
$\tilde{\tilde{n}}_{31}$	0.11	0.33	0.44	1.36	1.00	1.00
$\tilde{\tilde{n}}_{32}$	0.00	0.05	0.09	0.50	1.00	1.00
$\tilde{\tilde{n}}_{33}$	0.02	0.13	0.20	0.79	1.00	1.00
$\tilde{\tilde{n}}_{34}$	0.10	0.28	0.38	1.21	1.00	1.00
$\tilde{\tilde{n}}_{35}$	0.00	0.05	0.09	0.50	1.00	1.00
	$\tilde{n}^L_{kij}$					
	n^L_{1ij}	n^L_{2ij}	n^L_{3ij}	n^L_{4ij}	$H_1(\tilde{n}^L_{ij})$	$H_2(\tilde{n}^L_{ij})$
$\tilde{\tilde{n}}_{11}$	0.13	0.16	0.19	0.23	0.90	0.90
$\tilde{\tilde{n}}_{12}$	0.20	0.22	0.25	0.29	0.90	0.90
$\tilde{\tilde{n}}_{13}$	0.12	0.15	0.17	0.21	0.90	0.90
$\tilde{\tilde{n}}_{14}$	0.18	0.21	0.24	0.28	0.90	0.90
$\tilde{\tilde{n}}_{15}$	0.16	0.19	0.21	0.25	0.90	0.90
$\tilde{\tilde{n}}_{21}$	0.11	0.18	0.24	0.39	0.90	0.90
$\tilde{\tilde{n}}_{22}$	0.07	0.12	0.16	0.27	0.90	0.90
$\tilde{\tilde{n}}_{23}$	0.18	0.26	0.34	0.52	0.90	0.90
$\tilde{\tilde{n}}_{24}$	0.04	0.08	0.12	0.21	0.90	0.90
$\tilde{\tilde{n}}_{25}$	0.14	0.22	0.29	0.45	0.90	0.90
$\tilde{\tilde{n}}_{31}$	0.21	0.33	0.44	0.68	0.90	0.90
$\tilde{\tilde{n}}_{32}$	0.02	0.05	0.09	0.19	0.90	0.90
$\tilde{\tilde{n}}_{33}$	0.06	0.13	0.20	0.34	0.90	0.90
$\tilde{\tilde{n}}_{34}$	0.18	0.28	0.38	0.60	0.90	0.90
$\tilde{\tilde{n}}_{35}$	0.02	0.05	0.09	0.19	0.90	0.90

For example, the number 0.10 in Table 18 is calculated from the following equation:

$$0.10 = X_{11} \oslash (\tilde{\tilde{X}}_{11} \oplus \tilde{\tilde{X}}_{12} \oplus \tilde{\tilde{X}}_{13} \oplus \tilde{\tilde{X}}_{14} \oplus \tilde{\tilde{X}}_{15})$$

Table 19 shows the weighted normalization matrix ($\tilde{\tilde{E}}$) data obtained from Equations16 and 17:

Table 19. Normalized weight decision matrix ($\tilde{\tilde{E}}$)

	$\tilde{e}^U_{kij}$					
	e^U_{1ij}	e^U_{2ij}	e^U_{3ij}	e^U_{4ij}	$H_1(\tilde{e}^U_{ij})$	$H_2(\tilde{e}^U_{ij})$
$\tilde{\tilde{e}}_{11}$	0.08	0.15	0.30	0.30	1.00	1.00
$\tilde{\tilde{e}}_{12}$	0.13	0.21	0.36	0.36	1.00	1.00
$\tilde{\tilde{e}}_{13}$	0.07	0.14	0.28	0.28	1.00	1.00
$\tilde{\tilde{e}}_{14}$	0.12	0.20	0.36	0.36	1.00	1.00
$\tilde{\tilde{e}}_{15}$	0.11	0.18	0.32	0.32	1.00	1.00
$\tilde{\tilde{e}}_{21}$	0.02	0.10	0.61	0.61	1.00	1.00
$\tilde{\tilde{e}}_{22}$	0.01	0.07	0.44	0.44	1.00	1.00
$\tilde{\tilde{e}}_{23}$	0.04	0.15	0.77	0.77	1.00	1.00
$\tilde{\tilde{e}}_{24}$	0.01	0.05	0.36	0.36	1.00	1.00
$\tilde{\tilde{e}}_{25}$	0.03	0.13	0.69	0.69	1.00	1.00
$\tilde{\tilde{e}}_{31}$	0.07	0.26	1.31	1.31	1.00	1.00
$\tilde{\tilde{e}}_{32}$	0.00	0.04	0.48	0.48	1.00	1.00
$\tilde{\tilde{e}}_{33}$	0.01	0.10	0.76	0.76	1.00	1.00
$\tilde{\tilde{e}}_{34}$	0.06	0.22	1.17	1.17	1.00	1.00
$\tilde{\tilde{e}}_{35}$	0.00	0.04	0.48	0.48	1.00	1.00
	$\tilde{e}^L_{kij}$					
	e^L_{1ij}	e^L_{2ij}	e^L_{3ij}	e^L_{4ij}	$H_1(\tilde{e}^L_{ij})$	$H_2(\tilde{e}^L_{ij})$
$\tilde{\tilde{e}}_{11}$	0.12	0.15	0.22	0.22	0.90	0.90
$\tilde{\tilde{e}}_{12}$	0.18	0.21	0.28	0.28	0.90	0.90
$\tilde{\tilde{e}}_{13}$	0.11	0.14	0.21	0.21	0.90	0.90
$\tilde{\tilde{e}}_{14}$	0.17	0.20	0.28	0.28	0.90	0.90
$\tilde{\tilde{e}}_{15}$	0.15	0.18	0.25	0.25	0.90	0.90
$\tilde{\tilde{e}}_{21}$	0.05	0.10	0.26	0.26	0.90	0.90
$\tilde{\tilde{e}}_{22}$	0.03	0.07	0.18	0.18	0.90	0.90
$\tilde{\tilde{e}}_{23}$	0.08	0.15	0.34	0.34	0.90	0.90
$\tilde{\tilde{e}}_{24}$	0.02	0.05	0.14	0.14	0.90	0.90
$\tilde{\tilde{e}}_{25}$	0.07	0.13	0.30	0.30	0.90	0.90
$\tilde{\tilde{e}}_{31}$	0.16	0.26	0.61	0.61	0.90	0.90
$\tilde{\tilde{e}}_{32}$	0.02	0.04	0.17	0.17	0.90	0.90
$\tilde{\tilde{e}}_{33}$	0.05	0.10	0.31	0.31	0.90	0.90
$\tilde{\tilde{e}}_{34}$	0.13	0.22	0.54	0.54	0.90	0.90
$\tilde{\tilde{e}}_{35}$	0.02	0.04	0.17	0.17	0.90	0.90

For example, the number 0.08 in Table 19 is calculated from 0.08=$\tilde{n}_{11} \otimes \tilde{\tilde{w}}_{11}$Equation. Based on Table 19 and Equations 18 and 19, the values $\tilde{\tilde{S}}_{+i}$ and $\tilde{\tilde{S}}_{-i}$ are calculated as follows:

Table 20. $\tilde{\tilde{S}}_{+i}$values

	$\tilde{S}^U_{+i}$					
	S^U_{1+i}	S^U_{2+i}	S^U_{3+i}	S^U_{4+i}	$H_1(\tilde{S}^U_{+i})$	$H_2(\tilde{S}^U_{+i})$
$\tilde{\tilde{S}}_{+1}$	0.09	0.37	1.92	1.92	1.00	1.00
$\tilde{\tilde{S}}_{+2}$	0.01	0.11	0.93	0.93	1.00	1.00
$\tilde{\tilde{S}}_{+3}$	0.05	0.25	1.53	1.53	1.00	1.00
$\tilde{\tilde{S}}_{+4}$	0.07	0.27	1.54	1.54	1.00	1.00
$\tilde{\tilde{S}}_{+5}$	0.03	0.17	1.17	1.17	1.00	1.00
	$\tilde{S}^L_{+i}$					
	S^L_{1+i}	S^L_{2+i}	S^L_{3+i}	S^L_{4+i}	$H_1(\tilde{S}^L_{+i})$	$H_2(\tilde{S}^L_{+i})$
$\tilde{\tilde{S}}_{+1}$	0.21	0.37	0.87	0.87	0.90	0.90
$\tilde{\tilde{S}}_{+2}$	0.05	0.11	0.36	0.36	0.90	0.90
$\tilde{\tilde{S}}_{+3}$	0.13	0.25	0.65	0.65	0.90	0.90
$\tilde{\tilde{S}}_{+4}$	0.15	0.27	0.68	0.68	0.90	0.90
$\tilde{\tilde{S}}_{+5}$	0.08	0.17	0.47	0.47	0.90	0.90

For example, the number 0.09 in Table 20 is calculated from 0.09=$\tilde{\tilde{e}}_{21} \otimes \tilde{\tilde{e}}_{31}$equation, and, the number 0.08 in Table 21 is calculated from 0.08=$\tilde{\tilde{e}}_{11}$.

Table 21. $\tilde{\tilde{S}}_{-i}$values

	$\tilde{S}^U_{-i}$					
	S^U_{1-i}	S^U_{2-i}	S^U_{3-i}	S^U_{4-i}	$H_1(\tilde{S}^U_{-i})$	$H_2(\tilde{S}^U_{-i})$
$\tilde{\tilde{S}}_{-1}$	0.08	0.15	0.30	0.30	1.00	1.00
$\tilde{\tilde{S}}_{-2}$	0.13	0.21	0.36	0.36	1.00	1.00
$\tilde{\tilde{S}}_{-3}$	0.07	0.14	0.28	0.28	1.00	1.00
$\tilde{\tilde{S}}_{-4}$	0.12	0.20	0.36	0.36	1.00	1.00
$\tilde{\tilde{S}}_{-5}$	0.11	0.18	0.32	0.32	1.00	1.00
	$\tilde{S}^L_{-i}$					
	S^L_{1-i}	S^L_{2-i}	S^L_{3-i}	S^L_{4-i}	$H_1(\tilde{S}^L_{-i})$	$H_2(\tilde{S}^L_{-i})$
$\tilde{\tilde{S}}_{-1}$	0.21	0.37	0.87	0.87	0.90	0.90
$\tilde{\tilde{S}}_{-2}$	0.05	0.11	0.36	0.36	0.90	0.90
$\tilde{\tilde{S}}_{-3}$	0.13	0.25	0.65	0.65	0.90	0.90
$\tilde{\tilde{S}}_{-4}$	0.15	0.27	0.68	0.68	0.90	0.90
$\tilde{\tilde{S}}_{-5}$	0.08	0.17	0.47	0.47	0.90	0.90

The values of the following type 2 fuzzy numbers are defuzzified using the center point method presented inGhorabaee et al. [1]. Based on Tables 20 and 21 and Equation 20table 22 shows the values of $\tilde{\tilde{S}}_{+i}$ and $\tilde{\tilde{S}}_{-i}$ ranking calculation.

Table 22. $\tilde{\tilde{S}}_{+i}$ and $\tilde{\tilde{S}}_{-i}$ ranking values

i	$Rank\left(\tilde{\tilde{S}}_{+i}\right)$	$Rank\left(\tilde{\tilde{S}}_{-i}\right)$
1	0.32	0.32
2	0.30	0.32
3	0.32	0.31
4	0.32	0.32
5	0.31	0.32

For example, the number 0.32 in Table 22 is calculated from the following equation:

0.32=$\boldsymbol{Rank}\left(\tilde{\tilde{S}}_{+i}\right) = \frac{1}{20}(1+1+1+1+1)$,

0.32=$\boldsymbol{Rank}\left(\tilde{\tilde{S}}_{-i}\right)$=$\frac{1}{20}(1+0.93+1+0.94+0.97+1.5)$

By using equations21 and 22 the relative importance and usefulness of the alternatives are:

Table 23. $Q_i$and $U_i$values

i	Q_i	U_i
1	0.65	100
2	0.61	95
3	0.65	100
4	0.64	98
5	0.63	97

According to Table 23, the optimal ranking of options is as follows:

$$A_1 = A_3 \succ A_4 \succ A_5 \succ A_2 \quad (25)$$

The above results show that, according to the presented method in this article, the third supplier was the optimal choice, however after removing the effect of experts' credibility, the first supplier is selected as the optimal choice.

5. Sensitivity analysis

In this section, the sensitivity analysis results are presented. The following tables show the value of the *U* parameter and the ranking of the alternatives obtained by the COPRAS method. This is after changing the experts' credibility. The *R* numbers in these tables are coefficients that have been multiplied by the credibility of one of the decision-makers at each stage of sensitivity analysis.

Table 24. Analysis of the sensitivity of the optimal response to the change in the first expert's credibility

$d(\tilde{\tilde{y}}_{1j}) = R \times \left(Initial\ d(\tilde{\tilde{y}}_{1j})\right)$							
R=2		***R=4***		***R=6***		***R=8***	
U	*R*	*U*	*R*	*U*	*R*	*U*	*R*
98.6	2	98.8	2	98.8	2	98.8	2
94.9	5	95.9	5	96.5	5	96.8	5
100	1	100	1	100	1	100	1
98.3	3	98.4	3	98.6	3	98.7	3
96.2	4	96.8	4	97.1	4	97.3	4

Table 25. Analysis of the sensitivity of the optimal response to the change in the second expert's credibility

$d(\tilde{\tilde{y}}_{2j}) = R \times (Initial\ d(\tilde{\tilde{y}}_{2j}))$							
***R*=2**		***R*=4**		***R*=6**		***R*=8**	
U	*R*	*U*	*R*	*U*	*R*	*U*	*R*
99.3	2	99.8	2	100	1	100	1
94.3	5	94.8	5	95	5	95	5
100	1	100	1	99.9	2	99.6	2
98.6	3	98.7	3	98.7	3	98.4	3
96.1	4	96.6	4	96.8	4	96.9	4

Table 26. Analysis of the sensitivity of the optimal response to changes in the third expert's credibility.

$d(\tilde{\tilde{y}}_{3j}) = R \times (Initial\ d(\tilde{\tilde{y}}_{3j}))$							
R=2		***R=4***		***R=6***		***R=8***	
U	*r**	*U*	*r*	*U*	*r*	*U*	*r*
98.8	2	98.7	2	98.6	2	98.6	2
92.8	5	91.4	5	90.6	5	90	5
100	1	100	1	100	1	100	1
98.3	3	98.1	3	98	3	97.9	3
95.5	4	95.3	4	95.3	4	95.3	4

**r=Rank*

As the above tables show, by changing this parameter, the values of U change, but this change often does not affect the ranking of the alternatives. This shows that the experts' opinion about the rating of alternatives is largely similar. Of course, by multiplying expert number 2's credit by 6, the order of the options is also changed. The rank of the first and third alternatives is replaced.

The following tables show the ranking results of the COPRAS method after changing the average weight of the criteria. R numbers in these tables are coefficients multiplied by the average weight of one of the criteria at each stage of sensitivity analysis.

Table 27. Analysis of the sensitivity of the optimal response to changes in the average weight of the first criterion.

$d(\tilde{\tilde{\bar{w}}}_1) = R \times (Initial\ \tilde{\tilde{\bar{w}}}_1)$							
R=0.8		**R=0.6**		**R=0.4**		**R=0.2**	
U	r	U	r	U	r	U	r
98.9	2	98.9	2	98.9	2	98.9	2
94	5	94	5	94	5	94.0	5
100	1	100	1	100	1	100.0	1
98.4	3	98.4	3	98.4	3	98.4	3
95.9	4	95.9	4	95.9	4	95.9	4

Table 28. Analysis of the sensitivity of the optimal response to changes in the average weight of the second criterion.

$d(\tilde{\tilde{\bar{w}}}_2) = R \times (Initial\ \tilde{\tilde{\bar{w}}}_2)$							
R=0.8		***R=0.6***		***R=0.4***		***R=0.2***	
U	*R*	*U*	*R*	*U*	*R*	*U*	*R*
99.2	2	99.5	2	99.8	2	100	1
94	5	94	5	94.1	5	94.1	5
100	1	100	1	100	1	99.7	2
98.7	3	99	3	99.5	3	99.7	3
95.7	4	95.5	4	95.3	4	94.6	4

Table 29. Analysis of the sensitivity of the optimal response to changes in the average weight of the third criterion.

$d(\tilde{\tilde{\bar{w}}}_3) = R \times (Initial\ \tilde{\tilde{\bar{w}}}_3)$							
R=0.8		***R=0.6***		***R=0.4***		***R=0.2***	
U	*R*	*U*	*R*	*U*	*R*	*U*	*R*
98.6	2	98.5	2	98.3	2	98.1	2
94	5	94.2	5	94.5	5	95.3	5
100	1	100	1	100	1	100	1
98.1	3	97.7	3	97.2	3	96.1	3
96	4	96.4	4	97	4	97.5	4

As the tables above show, changing this parameter affects the *U* values, but this change often does not affect the ranking of the alternatives. This shows that the scores of different criteria are similar. Of course, when the weight of the second criterion is greatly reduced and multiplied by 0.2, the order of the alternatives is also changed. In addition, the rank of the first and third alternatives is replaced.

6. Discussion

Using the group decision-making approaches without evaluating the experts' know-how can lead to inappropriate results. This paper introduces a new multicriteria group decision-making approach based on the COPRAS method for selecting a sustainable supplier. The proposed method, in addition, to usingthe data and opinions of experts, also provides the possibility of applying the evaluation of the experts' know-how in any criteria. First, the experts express their opinions about the alternatives' scores and criteria weights using linguistic variables. The analyst then transferred them to the appropriate type 2 fuzzy numbers. The use of type 2 fuzzy numbers allows a better expression of the views of a group of experts because it presents a range of differences among experts' opinions. Also, the analyst gives his point of view about the validity of each expert's opinion in each criterion separately by a linguistic variable transformed into an intuitionistic fuzzy number. Using fuzzy intuitionistic numbers provides the possibility of presenting the hesitance of the analyst in his evaluation. The analyst may analyze the previous experts' views on the desired criteria. But if the analyst still cannot express with certainty the amount of knowledge of experts, he can show his doubt by using intuitionistic fuzzy numbers.

The difference between the results and the previous methods is shown by comparing the suggested method results and the original method for a hypothetical example. On the one hand, the required data increases in the proposed method; because the analyst has to evaluate the experts' know-how and, on the other hand, provides new possibilities for improving group decision-making efficiency. The first limitation of the research is the strong dependence of the results on the analyst's opinion because the coefficients assigned by the analyst highly affect the impact of expert opinions. Determining the credibility of experts in each criterion as a qualitative parameter may face many challenges. Personal judgments, desires, or other factors may affect the analyst's opinion. Therefore, one of the critical studies domains in the future is calculating the credibility of each expert supported by sound reasons and based on evidence.

Another limitation of this study is gathering more information for the decision model.The analyst may apply the experts' credibility in the model in a different way. In this study, it is multiplied by the standard weight, and thus it linearly affects the suppliers' scores. Using the other methods gives different results whose validation should be discussed.

To confirm the proposed model, its results were compared with the results of other MCDM models including EDAS, MARCOS, and TOPSIS. The rank obtained from these models and the correlation coefficient (Spearman ranking correlation) between the proposed model and other models are as follows:

Table 30. Table 1: Comparison between the results of the proposed model and other MCDM models and other MCDM models

	COPRAS	TOPSIS	MARCOS	EDAS
A1	2	3	2	2
A2	5	2	3	4
A3	1	1	1	1
A4	3	5	4	3
A5	4	4	5	5
τ_s		0.6	0.7	0.9

Spearman ranking correlation is calculated by the following equation:

$$\tau_s = 1 - \frac{6\sum(d_i)^2}{n(n^2-1)} \tag{26}$$

Where τ_s, d_i and n are Spearman's rank correlation coefficient, the difference between the two ranks of each model, and the number of alternatives. The table above shows that the results of the proposed method are the closest to the results of the EDAS method, followed by the MARCOS and TOPSIS methods.

Conclusion

In this article, a new approach is presented to consider the difference in the validity of experts in commenting on the score of each option in each of the criteria, in the group COPRAS method. By presenting a numerical example, the method of this method to select a sustainable supplier is shown. By changing the main parameters, the sensitivity of the model response to them has been determined and the results have been discussed. The proposed method can be used to increase the validity of the group decision-making method.

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