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Obstacle Avoidance and Travel Path Determination in Facility Location Planning

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Abstract

This research finds a solution methodology for determining optimal travel path to and from existing facilities and corresponding location of a new facility having physical flow interaction between them in different degrees translated into associated weights, in presence of barriers impeding the shortest flow-path involving straight-line distance metric. The proposed methodology considers all types of quadrilateral barriers or forbidden region configurations to generalize, to by-pass these obstacles, and adopts a scheme of searching through the vertices of these quadrilaterals to determine the alternative shortest flow-path for optimal location of facilities based on weighted-distance computation algorithm with minimum summation or mini-sum objective. Congruence testing has been carried out for reconfiguring complex obstacle geometry as an equivalent quadrilateral. This procedure of obstacle avoidance is completely new. Software, DANSORK, has been developed to facilitate computations for the new search algorithm and test results have been presented based on computations using this software.

Keywords: New Facility Location; Obstacle Avoidance; Quadrilateral Barrier Configuration; Computational Software

1. Introduction

The problems of locating a new manufacturing plant, or a distribution warehouse, and similar other facilities always involve the consideration of transport costs, which is dependent on the distance traveled for moving materials between such new facility and other existing facilities having logistic interactions such as; raw materials depot, feeder factory, or customer's warehouse. The distances between new facility and each of the existing facilities determine the optimal new facility location such that the total transportation cost is kept to minimum. The quantum and frequency of material movement load or the weight influences the considerations for fixing distances. Hence, optimality is dependent on weight, which is the product of the cost per unit distance of travel and frequency of trips per unit time period. The distance between facilities for material flow has been considered on the Euclidean distance metric with minimum summation or mini-sum objective. In many real life situations, the straight path for material flow between facilities is not available due to presence of forbidden regions or barriers such as; protected land, lake, another plant ,or any other physical obstacle creating constrained condition for movement. There has been sustained research interest in the area of facility location, which is reflected through the review of literature presented in the following paragraphs.

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Contreras and Diaz [1] considered Single Source Capacitated Facility Location Problem and proposed a Scatter Search approach. Also, a tabu search algorithm was applied. It has been observed that the method provides solutions with reasonable computational effort. Online Facility Location problem was dealt with by Fotakis [2], where the demand points arrive online, which must be assigned irrevocably to an open facility upon arrival. The objective is to minimize assignment costs. A multi-stage facility location problem, in the context of supply chain, formulated as a mixed integer program, has been presented by Wollenweber [3] where a two-phase heuristic solution approach was adopted. The greedy construction heuristic utilizes the solution obtained by the LP-relaxation of the problem. A study [4] addressed an evaluation of new heuristics solution procedures for the location of cross-docks and distribution centers in supply chain network design, where the model is characterized by multiple product families, a central manufacturing plant site, multiple cross-docking and distribution center sites, and retail outlets that demand multiple units of several commodities. A three-tier distribution network was examined [5]. It consists of a single supplier at a given location, a single intermediate warehouse whose location is to be determined, involving multiple retailers at given locations. Berman et al. [6] considered the problem of locating a set of facilities on a network to maximize the expected number of captured demand when customer demands are stochastic and congestion exists at facilities and propose heuristic-based solution procedures. A facility location problem in a continuous planar region considering the interaction between the facility and the existing demand points, and it has been dealt with by Karasakal and Nadirler [7] to maximize the weighted distance of the facility from the closest demand point as well as to minimize the service cost of the facility while the distance between the facility and the demand points was measured with the rectilinear metric. An interactive branch and bound algorithm were suggested to find the most preferred efficient solution. Four variations of the ant colony optimization meta-heuristic that explore different construction modeling choices were developed by Randall [8] in the context of capacitated hub location problem, and the results of the study reveal that the approaches can provide optimal solution in a reasonable amount of computational time. The work done by Dias et al. [9] describes a new multi-objective interactive algorithm and its ability to solve capacitated and uncapacitated multi-objective single, or multi-level dynamic location problems. The algorithm is part of an interactive procedure that asks the decision maker to define interesting search areas by establishing limits to the objective function values, or by indicating reference points. A new implementation of a widely used swap-based local search procedure for the p-median problem was presented by Resende and Werneck [10], and they have shown that their method can be adapted to handle the facility location problem and to implement related procedures like path-relinking and tabu search. Villegas et al. [11] modeled a bi-objective (cost-coverage) incapacitated facility location problem and designed and implemented three different algorithms that were able to obtain a good approximation of the Pareto frontier. The work of Farhan and Murray [12] developed general models that simultaneously address issues involving potential demand as a function of distance, coverage range, and partial regional service in facility siting. The developed models are general since they can be utilized for siting both desirable and undesirable facilities. Jia et al. [13] analyzed the characteristics of large-scale emergencies and proposed a general facility location model that is suited for large-scale emergencies where the general facility location model can be cast as a covering model, a *P*-median model or a *P*-center model, each suited for different needs in a large-scale emergency. Berman et *al.* [14] analyzed the problem of locating a set of service facilities on a network when the demand for service is stochastic and congestion may arise at the facilities while considering two potential sources of lost demand like increasing travel distance and long queues. They were investigated through several integer programming formulations and heuristic approaches in this context. A genetic-like algorithm was proposed by Pelegrín et al. [15] which is able to find a predetermined number of global optima, if they exist, for a variety of discrete location problems. Berman and Krass [16] examined the incapacitated facility location problem with a special structure of the objective function coefficients where for each customer the set of potential locations can be partitioned into subsets such that the objective function coefficients in each are identical. This structure exists in many applications, including the Maximum Cover Location Problem. Angelopoulos and Borodin [17] applied the priority algorithm framework to define "greedy-like" algorithms for the uncapacitated facility location problems and set cover problems. Drezner [18] solved gravity model for the competitive facility location problem and have shown that the generalized Weiszfeld procedure converges to a local maximum, or a saddle point while also devising a global optimization procedure that finds the optimal solution within a given accuracy. In a study Shaw [19] showed that several well-known facility location problems can be formulated uniformly into a special structured Tree Partitioning Problem; and also developed a generic algorithm to solve such facility location problems. In a study Berman and Drezner [20] investigated the problem of locating a given number of facilities on a network where demand generated at a node is distance dependent, and the facilities can serve no more than a given number of customers. Models proposed (Zhou and Liu, 2007) [21] for capacitated location–allocation problem with fuzzy demands, and several numerical experiments have been presented to illustrate the efficiency of some proposed algorithms.

It is evident from the aforementioned review that there are not many solution procedures for handling such location search problems involving barriers or forbidden areas in any number and shape which are present between facilities impeding straight path between them. However, in the recent past, the facility location problems involving barriers or forbidden regions have drawn the attention of the researchers in this area. Aneja, et al. [22] dealt with the barriers and forbidden regions based on network formation approach in location problems while Batta, et al. [23] proposed a solution with an approach of cell formation. Eckhardt, as mentioned by Katz et al. [24], dealt with some problems involving forbidden regions with polygonal configuration in which the paths are allowed through the forbidden region, but prohibiting the location of facility within the region. They studied the problem of single facility location involving Euclidean distance metric with mini-sum objective. Brady et al. [25] deployed interactive graphics to solve facility location problems with a minimax objective function involving single as well as multiple new facilities in presence of forbidden region having any arbitrary configuration. . Hamacher and Nickel [26] studied the location problem involving restrictions of forbidden region for developing the solution algorithms for median problems in the plane. Bypassing impenetrable barriers for placement of a single finite-size facility following rectilinear norm has been addressed by Savas et al. [27]. This interactive model considered identification of candidates for optimal placement for a facility with a fixed orientation, and then for a facility with a fixed server location and presented a heuristic. Optimal location of facilities in presence of impenetrable barriers following rectilinear metric has been attempted by Larson and Sadiq [28]. The new facility location for planar 1-median problem with convex polygonal forbidden regions has been addressed by McGarvey and Cavalier [29], and a solution procedure using the 'Big Square Small Square' branch-and-bound is developed for global optimization. Most of the aforementioned studies consider either single forbidden region, or any specific shape of the restricted region. The objective of the present study, therefore, is to formulate a single facility location model amidst a host of existing facilities adhering Euclidean distance norm and restricted by single or multiple forbidden regions. The configuration of forbidden barriers in most of the studies is considered to be either rectangular or circular. The model presented in this paper is generalized in the sense that it considers the forbidden barriers in multiple numbers with arbitrary quadrilateral shapes including rectangle or irregular polygon to cover most of the applications using a single solution framework. A treatment is proposed here to reconfigure complex barrier geometry as an equivalent quadrilateral, and a congruence test has been carried out. In order to bypass the barrier contour and establish a flow route between facilities, a Cartesian grid element search method or cell formation or network formation approach wasdeployed earlier for exploring the alternative flow path. But such method is inefficient to handle multiple barriers of different geometries obstructing the flow path; and cannot be deployed correctly for determining the constrained shortest flow path. This research adopts a completely new approach in formulating the least-path search through the vertices of forbidden barriers and in developing an appropriate algorithmic computational methodology for a single facility location problem. Necessary generalization has also been made by way of considering the constraints with multiple barriers with arbitrary quadrilateral contour. The solution software,DANSORK, which has been developed[30] for determining the optimal location of a new facility under both constrained as well as unconstrained situations, will run on a simple PC.

2. Distance Computation Involving Quadrilateral Barriers

2.1. Distance Computation Involving Quadrilateral Barriers

The minimum distance between existing and new facility in presence of an quadrilateral obstacle as shown in Figure1 (A) & (B) has been either the path connecting the points (x_e, y_e) ; (x_2, y_2) ; (x_m, y_m) , that is through one vertex point of the barrier quadrilateral. Where the distance,

$$d_1 = [(x_e- x_2)^2 + (y_e- y_2)^2]^{1/2} + [(x_2- x_m)^2 + (y_2- y_m)^2]^{1/2} \quad (1)$$

Or, through the path, connecting the points (x_e, y_e); (x_4, y_4); (x_3, y_3); (x_m, y_m) , that is through two adjoining vertices of the barrier quadrilateral. Where the distance,

$$d_2 = [(x_e- x_4)^2+ (y_e- y_4)^2]^{1/2} + [(x_4- x_3)^2+ (y_4- y_3)^2]^{1/2} + [(x_3- x_m)^2+(y_3- y_m)^2]^{1/2} \quad (2)$$

Minimum of d_1 or d_2 is the shortest path between an existing facility at a fixed and the new facility at any arbitrary location. Any of the two paths will be treated as shortest where $d_1 = d_2$.

To obviate the complexity in computation for the second case, that is, for computation of path length through two adjoining vertices of the barrier, a simplified distance computation procedure with a degree of approximation has been adopted, which will produce reasonably accurate results in practice. The distance in this case is the summation of two distance segments, namely, the distance from the respective existing facility to the nearest vertex of the obstructing quadrilateral and from the same vertex to the new facility located at any arbitrary point. This is the approximate substitution of the combination of three distance segments, namely, the distance of the existing facility from the nearest vertex of the obstructing quadruple, the distance of new facility from its nearest vertex and the distance between these two vertices. The justification is supported experimentally with three hundred problem samples. Most of the distance computations, as derived from the simulated experiment, are oriented with the involvement of single vertex of the barrier quadrilateral where the need of such approximation is absent altogether while in fewer other cases, the distance computations involve consideration of the adjoining vertices where the aforementioned approximation will be necessary. Thereby the overall effect of such approximation error is minimized, and in fact, is within one percentage as has been observed in the results of this experiment.

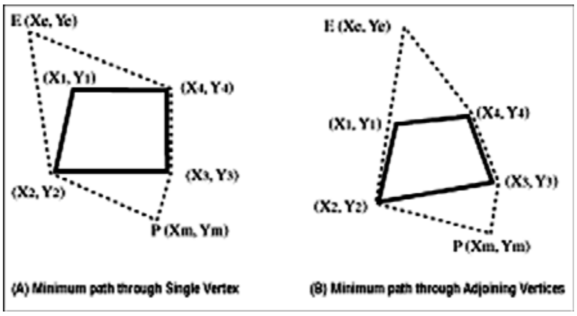

Figure 1: Least path Distance Search Scheme For Quacrilateral Barrier, a, b.

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2.2. Reconfiguration of a Complex Barrier as Equivalent Quadrilateral

The basic solution procedure involved in the analytical software based technique, as has been developed for the present research, does consider the forbidden regions or barriers having quadrilateral configuration only in a Euclidean space. Such quadrilaterals, of any form, can be immediately configured into the model without having to make any approximation in the geometry of the contour, unlike few other available techniques where only rectangular quadrilateral feature can be used without a contour approximation. Obviously, and also as reflected from the review of past research, such contour approximation is even more cumbersome, if possible at all, for other complex configurations such as geodesic, polygon, or composite shapes when present in a location analysis problem. However, since situations are encountered involving such complex forbidden configurations in many real-life facility location problems, an attempt has been made in the current research to develop some new techniques by which these complex or irregular obstacle configurations can also be accommodated into the location modeling framework while keeping the procedure still quite simple. In order to achieve this objective, the inter-facility distances have been simulated by transposition of an equivalent quadrilateral in substitution of the original complex configuration. A technique, with a simplistic approach, has been designed for deriving the configurationally equivalent quadrilaterals based on the parametric considerations as elaborated in the subsequent paragraphs.

The basic premise, that any uni-planar configuration will have extreme or boundary points along x-axis and y-axis in a Cartesian plane and eventually,

1. two such points, will be transfixed by x-axis; while
2. other two will be transfixed by y-axis.

Thus, such considerations and the corresponding treatment will give rise to four such extreme points in all. Hence, a quadrilateral will be formed by connecting these four strategic points which can be treated as an equivalent transposition, for example in the case of an inscribed square within a circle or within a polygon with the shape of a regular octagon. Another modality in framing the equivalent quadrilateral can be adopted by way of:

1. Projecting parallel lines to x-axis through the two extreme boundary points located on y-axis; and
2. Projecting parallel lines to y-axis through the two extreme boundary points located on x-axis.

Now, the extension of four such lines will produce four intersection points. A quadrilateral will be formed by connecting these four strategic points which can be treated as an equivalent transposition, like in the case of a circumscribed square on a circle or on a polygon with the shape of a regular octagon. However, while framing such equivalent quadrilaterals, extreme conditions may be generated where such configurations will be either completely inscribed in or circumscribed on the actual geometric contour as have been mentioned in the above cases, involving a geodesic (circular) and a polygon forbidden region. This can further be refined by moderating the construction technique for the equivalent configuration. This would be moderated as the mean of the above equivalent rectangular quadrilaterals, a square shaped in this particular case, for both the inscribed as well as circumscribed construction. This has been treated as the rationalized equivalent for all practical purposes. Such intermediate or moderated quadrilateral can be formed with the average or mean geometrical dimensions. Another rationalization modality can also be considered for construction, as has been adopted in the present casefor setting the equivalence. This has been done by selecting the mid points of a segment(s) parallel to any or both Cartesian axes as the one represented in Figure1C. Very often in such constructions, a part of the actual configuration remains outside the boundary of the equivalent quadrilateral where at the same time some portion of the actual contour is inside the contour of the equivalent configuration. The effect on distance computations due to such Transfiguration of obstacle geometries to equivalent quadrilateral shapes has been examined and presented in the congruence analysis section involving four extreme cases, as highlighted before, along with a moderated case as depicted in Figure1C. The combinational overall average error in distance computation has been found to be within 1% based on a simulated analysis carried out with randomly selected population of facility location coordinate points. The results of the analysis clearly demonstrate the correctness and suitability of the equivalence approach of transfiguring complex geometrical obstacles into the equivalent quadrilaterals.

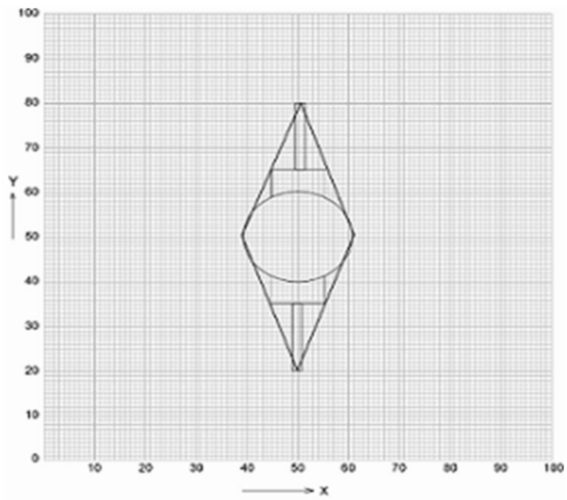

Figure 1c : Equivalent Quadrilateral for the Actual Configuration.

The validity testing for the degree of accuracy in distance computation between two points in presence of an equivalent quadrilateral, as a substitute for simulating the original complex configuration, has been analyzed and is presented in this section. This congruence testing is required to examine the practical usability of such transfigured equivalence. The procedural approach towards congruence analysis has been classified in two broad categories as follows based on the parametric treatment made on the actual geometric shape for transfiguration into an equivalent quadrilateral:

1. In the first category, the analysis has been carried out involving only extreme conditions where only completely inscribed or completely circumscribed equivalent quadrilaterals have been considered for a circular and a regular octagon. Whereas,

2. In the second category, detailed analysis has been carried out considering the composite type configuration as shown in Figure1C as a representative case.

It has been intended in the present set of analysis to determine the extent of deviation or error associated with such extreme cases because of the fact that if such errors are found to be within the acceptable limits for similar practical applications, then the usability of such techniques are far more cogent for the situation where the actual configuration is transposed with moderated equivalent quadrilaterals as depicted in Figure 1C. The actual obstacle configurations and the corresponding equivalents, based on already referred construction modalities, for both categories of equivalence, e.g., (i) extreme condition and (ii) moderated condition, have been elicited for the former and depicted in a graphical format for the latter for the dimensional and other related analysis where the problem space in question has been considered to be a planar matrix of hundred graphic units. The random number based location co-ordinate values have been expressed in graphical units for placing those in the matrix space while the forbidden configurations were transposed centrally for the congruence analysis as an example case has been presented in Figure 1C. The original complex geometry is depicted here with thin lines and the equivalent quadrilateral with thick lines in a 100 unit x 100 unit Cartesian grid space. The validity testing has been carried out to check the degree of accuracy in distance computation between two points in presence of an equivalent quadrilateral as a substitute for simulating the original complex configuration. This congruence testing is required to examine the practical usability of such transfigured equivalence.

The validity, or as may be called here as the congruence testing in the analysis of the above cases, has been carried out using a sizable three hundred (300) pairs of random number based co-ordinate values to represent the location points of the new and the existing facility.

The distance between the location points representing the facilities, in presence of forbidden configurations, has been either mathematically computed, or physically measured as was found appropriate in a particular quantification situation for onward comparison and also for determination of associated error. While the computation of the least path between two facilities through the single corner point of a quadrilateral shaped forbidden region has been relatively simple, an approximation technique with additional consideration of using a single vertex instead of two in distance computation as stated in the earlier section, had been necessary for the situations where the least intersection free path has been through two adjoining corner points of the forbidden quadrilateral. Based on this technique, a shorter-minimum path and a relatively longer path will be obtained. The summarized results and relevant comparative analysis in distance simulation with the extreme as well as the moderated equivalent quadrilateral construction for the corresponding true shape have been presented in TABLE-1 and TABLE-2 respectively. Orientation condition for congruence testing and analysis results for equivalent quadrilaterals simulating the actual forbidden configuration are based on the following structural pattern. The already referred selected configurations, e.g., the circle and the octagon, representing a geodesic and a polygon shape respectively, have been considered in the analysis of transposing extreme equivalent quadrilateral.

The select compositions for such analytical comparison are:

1. Inscribed and circumscribed quadrilateral within and on a circle separately; and
2. Inscribed and circumscribed quadrilateral within and on a polygon, a regular octagon here, in two separate sets.

The details of data analysis and deviation results for each individual set, as mentioned in the above sequential order, are presented in the TABLE – 1, and the case with the composite configuration and its equivalent quadrilateral are presented in TABLE-2. The solution procedure, here, does consider the barrier only with quadrilateral configuration. However, since many real-life barriers have rather complex configuration, attempt has been made in this paper to simplify the shape to fit in a modified or equivalent quadrilateral. It has been worked out in this fashion, so that other barrier shapes can also be included with some approximations in the solution format proposed here. The congruence or validity testing of such approach has been carried out. The effect of distance computations due to such transfiguration of complex obstacle geometry to equivalent quadrilateral shape has been analyzed with a sample case as shown in Figure1C. Random number based coordinate values, representing facility locations, have been expressed in grid-units for placing those in the Cartesian space while the barrier configurations were transposed centrally for the congruence analysis. The congruence testing has been carried out using a sizable three hundred (300) pairs of coordinate values generated based on random numbers to represent the location points of the new and the starting location facility through each such pair. The distance between each such pair of points representing facilities, bypassing the barrier configuration, has been computed analytically for comparison and to determine the error. The relevant analysis and summarized results in distance simulation with equivalent quadrilateral construction for the corresponding original shape have been presented in TABLE-2, which shows the ‘overall average percentage of error’ is to the tune of 0.25%. Such approximation yielding very low error is acceptable within practical limits, and hence has been accommodated in the current solution model.

3. A New Method for Distance Computation on Successive Identification of Barriers

3.1. Identification of Obstructive Quadrilateral

In order to establish a relationship between an existing facility and the corresponding obstructive quadrilateral, impeding a straight path to new facility, a mathematical identification of the particular barrier is necessary for iterative computation through computerized software. If a quadrilateral poses an obstruction on the straight line formed by joining the new and an existing facility, then logically the line has to intercept at least on two arms of that polygon.

Table 1 : AVERAGE VALUES AND FREQUENCIES (For Actual vis-à-vis Equivalent Configuration).

(1)	………………….(2)………………….	(3)	(4)	(5)	(6)	(7)	…(8)
1	Total test population of randomized strategic coordinate point sets(pairs) representing new facility and starting location facility	300	300	300	300	300	
2	Reference(Equivalent) configuration of 'Barrier'	I.S	C.S	I.S	C.S		
3	Frequency of occasions – the reference configuration posing obstruction on the straight line joining the pair of strategic points	66	83	45	70	66	45 - 83
4	Frequency of occurrence – the minimum distance is through single corner point(vertex) of the equivalent quadrilateral.	56	68	38	60	56	38 – 68
	(a) Average percentage of error in computing minimum distance through single corner point of the equivalent quadrilateral vis-à-vis the actual configuration.	0.53%	2.57%	0.76%	1.20%	1.27%	0.53%-2.57%
	(b) Largest percentage of error in computing minimum distance through single corner point of the equivalent quadrilateral vis-à-vis the actual configuration.	1.86%	11.17%	3.73%	5.03%	5.45%	1.86%-11.17%
5	Frequency of occurrence – the minimum distance is through adjoining two corner points(vertex) of the equivalent quadrilateral.	10	15	7	10	10.5	7 – 15
	(a) Average percentage of error in computing minimum distance through adjoining two corner points of the equivalent quadrilateral vis-à-vis the actual configuration.	3.93%	2.29%	1.68%	1.30%	2.3%	1.30%-3.93%
	(b) Largest percentage of error in computing minimum distance through adjoining two corner points of the equivalent quadrilateral vis-à-vis the actual configuration	14.88%	9.67%	3.25%	7.0%	8.7%	3.25%-14.88%
6	Grand average percentage of error in computing minimum distance through both the single and double point(s) oriented situations.	1.05%	2.52%	0.90%	1.21%	1.42%	0.90%-2.52%
7	Frequency of occurrence that the straight path connecting two strategic points are obstructed by the original obstacle configuration only and not by the superimposed equivalent quadrilateral.	5	-	15	-	5	NIL-15
	- Average percentage of error where the actual configuration poses obstruction only while the overlapping equivalent quadrilateral does not.	0.24%	-	0.29%	-	0.13%	NIL-0.29%
8	Frequency of occasions – the equivalent quadrilateral does not pose obstruction on the straight line segment connecting two strategic points including the cases where one or both strategic point(s) lie within the configuration(s)	229	217	240	230	229	217 – 240
9	Overall average percentage of error considering entire test population of strategic coordinate point sets.	0.23%	0.70%	0.14%	0.28%	0.34%	0.14%-0.70%

*Column No.(1): Serial Number.
*Column No.(2): Criterial Features.
*Column No.(3): Circular Configuration Vs. Inscribed Equivalent Quadrilateral.
*Column No.(4): Circular Configuration Vs. Circumscribed Equivalent Quadrilateral.
*Column No.(5): Regular Octagon (Polygon) Configuration Vs. Inscribed Equivalent Quadrilateral.
*Column No.(6): Regular Octagon (Polygon) Configuration Vs. Circumscribed Equivalent Quadrilateral.
*Column No.(7): Combinatorial Average Values.
*Column No.(8): Combinatorial Range of Values.

Table 2: AVERAGE VALUES AND FREQUENCIES (For Actual vis-à-vis Equivalent Configurations).

SL.No.	Criterial Feature	Values
1	Total test population of randomized strategic coordinate point sets(pairs) representing new facility and starting location facility	300
2	Reference(Equivalent) configuration of 'Barrier'	Superimposed overlapping quadrilateral
3	Frequency of occasions – the reference configuration posing obstruction on the straight line joining the pair of strategic points.	97
4	Frequency of occurrence – the minimum distance is through single corner point(vertex) of the equivalent quadrilateral. (a) Average percentage of error in computing minimum distance through single corner point of the equivalent quadrilateral vis-à-vis the actual configuration. (b) Largest percentage of error in computing minimum distance through single corner point of the equivalent quadrilateral vis-à-vis the actual configuration.	90 0.58 % 2.20 %
5	Frequency of occurrence – the minimum distance is through adjoining two corner points(vertex) of the equivalent quadrilateral. (a) Average percentage of error in computing minimum distance through adjoining two corner points of the equivalent quadrilateral vis-à-vis the actual configuration. (b) Largest percentage of error in computing minimum distance through adjoining two corner points of the equivalent quadrilateral vis-à-vis the actual configuration.	7 3.36 % 11.45 %
6	Grand average percentage of error in computing minimum distance through both the single and double point(s) oriented situations.	0.78 %
7	Frequency of occurrence that the straight path connecting two strategic points are obstructed by the original obstacle configuration only and not by the superimposed equivalent quadrilateral. Average percentage of error where the actual configuration poses obstruction only while the overlapping equivalent quadrilateral does not.	3 0.06 %
8	Frequency of occasions – the equivalent quadrilateral does not pose obstruction on the straight line segment connecting two strategic points including the cases where one or both strategic point(s) lie within the select configuration(s)	200
9	Overall average percentage of error considering entire test population of strategic coordinate point sets.	0.25 %

It is imperative to check mathematically as to whether the intercepting intersection points are on and within the polygon arm segment. The polygon would be treated as an obstacle if more than one such intersection points are obtained for any particular polygon. The mathematical equation of a polygon arm can be expressed in the general form as;

$$ax + by + c = 0 \qquad (3)$$

Where, x and y are cardinal variables; and a ,b ,c are coefficients. Such coefficient values [a; b; c] for each arm of a particular polygon connecting two vertices (x_s,y_s) and (x_t, y_t) would be given by:

$$[(y_t - y_s)/(x_t - x_s) \; ; \; -1 \; ; \; y_s - (y_t - y_s)/(x_t - x_s) * x_s]$$

Similarly, the coefficients $[a' \; ; \; b' \; ; \; c']$ for the line equation joining new facility location point (x_m, y_m) and each of the existing facility location point (x_e, y_e) would be given by:

$$[(y_m - y_e)/(x_m - x_e); \; -1 \; ; \; y_e - (y_m - y_e)/(x_m - x_e) * x_e]$$

The coefficients of line equations for each arm of all polygons as well as of lines joining the new facility and each of the existing facilities are computed using developed software. The aforementioned intersection points, x_{int} and y_{int} are derived as follows:

$$[\, x_{int} \; ; \; y_{int} \,] = [(c' - c)/(a - a') \; ; \; (c' a - a' c)/(a - a')] \qquad (4)$$

Subject to the following sets of conditions :

$(x_s \quad x_{int} \quad x_t)$ or $(x_s < x_{int} < x_t)$; while, $[y_{s(\text{ or } t)} \quad y_{int} \quad y_{t(\text{ or } s)}]$ or $[y_{s(\text{ or } t)} < y_{int} < y_{t(\text{ or } s)}]$ and

$(x_e \quad x_{int} \quad x_m)$ or $(x_e < x_{int} < x_m)$; while, $[y_{e(\text{ or } m)} \quad y_{int} \quad y_{m(\text{ or } e)}]$ or $[y_{e(\text{ or } m)} < y_{int} < y_{m(\text{ or } e)}]$

A polygon would be treated as obstacle for the particular existing facility provided that the above conditions are satisfied together. This procedure is repeated successively for all obstacles for every existing facility .The next step is to compute the minimum distance bypassing the polygon, in case the same has been identified as an obstacle; and is presented in the following section.

3.2. A Methodology for Computation of Minimum Distance in Presence of Obstacle

Lines joining new facility and all the four vertex points of the particular obstructive polygon would generate equations of two lines, those are tangent to the polygon at two vertices and two other lines intersect the polygon. The subsequent computational step is to identify a couple of tangent vertices out of all four in a polygon. This is accomplished by following similar procedure adopted for identification of obstacles. Here, the coefficients [a'' ; b'' ; c''] of a line equation joining a vertex (x_s, y_s) of the obstructive polygon and the new facility point(x_m, y_m) is given by,

$$[(y_m - y_s)/(x_m - x_s)\ ;\ -1\ ;\ y_s - (y_m - y_s)/(x_m - x_s)*x_s]$$

And the intersection point produced by this line with one of the polygon arms would be given by:

$$[x_{int'}\ ;\ y_{int'}] = [(c'' - c)/(a - a'')\ ;\ (c''a - a''c)/(a - a'')] \quad (5)$$

Subject to**;**

$(x_s \quad x_{int'} \quad x_t)$ or $(x_s < x_{int'} < x_t)$,while,

$[y_{s(\text{ or } t)} \quad y_{int'} \quad y_{t(\text{ or } s)}]$or $[y_{s(\text{ or } t)} < y_{int'} < y_{t(\text{ or } t)}]$

Where, s and t are the terminal points of the line. Any line joining one of the vertices and the new facility would intersect one of the arms of the polygon, in the case that the particular vertex is not a tangent vertex. Mathematically, in such case, the number of intersections computed would be three including two occasions where it is on the same point at the vertex which is the common point lying on two intersecting arms of the polygon. This common point, mathematically, is counted twice and therefore, in the case of a tangent,the number of such intersections is two. A counter for computed number of such intersections has been provided in this software. Now, by connecting the two tangent points of polygon to location point of the specific existing facility, two separate paths with computed distances following the approximation technique as referred in section (2), are obtained and a minimum of these is selected. The mathematical expression of minimum distance through the corresponding polygon vertex points (x_{sk}, y_{sk}), of the j^{th} obstacle at the k^{th} vertex, from the new facility point (x_m, y_m) to the existing i^{th} facility point (x_{ei}, y_{ei}) is given as below :

$$\text{Distance} = [(x_{jk} - x_m)^2 + (y_{jk} - y_m)^2]^{1/2} + [(x_{jk} - x_{ei})^2 + (y_{jk} - y_{ei})^2]^{1/2} \quad (6)$$

The summation of the products of the distances for both constrained and unconstrained situations and relative weight(w_i) associated with each existing facility is the total cost (C) burden for the particular location of the new facility which can be mathematically expressed as :

$$C= \ w_i\{[(x_{jk} - x_m)^2 + (y_{jk} - y_m)^2]^{1/2} + [(x_{jk} - x_{ei})^2 + (y_{jk} - y_{ei})^2]^{1/2}\} + \ w_i[(x_m - x_{ei})^2 + (y_m - y_{ei})^2]^{1/2} \quad \ldots\ldots\ldots\ldots\ldots\ldots(7)$$

Constrained, unconstrained .

The constrained condition arises in presence of barriers while in absence of barriers the condition is unconstrained.

3.3. Cardinally Explorative Search Procedure for Optimal Location Determination

In the new facility at any suboptimal or optimal situation, say at any arbitrary location, P can physically be any point within the spatial boundary of existing facilities, E_i located at different coordinate points in the cardinal plane and d* is the effective distance. For optimality searching, any coordinate point representing the new facility can be chosen as starting point from where the searching can begin. Assuming that the starting point is (x_0, y_0) for the new facility in the chosen plane having a corresponding value of cost burden (C) as C_0 .

$$\text{Where, } C = \ \overset{n}{\underset{e_i=1}{}} w_i\, d^*(P,E_I) \quad (8)$$

The next step is to check the value of C, 1-unit apart in all four cardinal directions (2 on the abscissa and 2 on the ordinate) namely at (x_0, y_0+1); (x_0+1, y_0); $(x_0, y_{0}-1)$; (x_0-1, y_0). Supposing that in the 1^{st} iteration, the minimum value at any of the above cardinal points is C_1. Then in the next iteration, the coordinate corresponding to the above minimum C (i.e, C_1) will be treated as the fresh starting point for the next iteration, and so on till the value of C converges to a minimum and the coordinate point (x_m, y_m), corresponding to such minimum value is the optimal location point of the new facility. This algorithmic technique, developed in the optimality modeling software with graphical representation, is oriented with a cardinal exploration based searching through converging series of locational values.

4. Salient Features of the Developed Analytical Software for Locational Optimality.

The construction of the software for analytical solutions of the location analysis problem has been constructed for graphical representation of the optimality framework that can run on a PC and is structured on integrated functional modules. Data Entry for various input of the basis are necessary for defining the problem conditions associated with a new facility location analysis, and pertaining to the coordinate location of facilities, obstacle locations, inter-facility load-flow weightages, and search iteration starting point.Based on the data entry sequence, these existing facilities are automatically numbered as E1, E2, E3, etc., while the quadruple obstacle vertices are numbered like F011, F012, F013, and F014, where the alphabet (F) denotes the forbidden barrier as the succeeding two digits represent the obstacle reference number while the third digit indicates each vertex number of that specific quadrilateral that needs to be keyed in a sequential order for defining the barrier configuration. The iteration starting reference coordinate point is also to be keyed in as the operating input data.

5. Experimental Results

An experimental sample problem-set involving six existing facilities under constrained condition with five forbidden barriers is presented in this section. The results of optimality search using the software are also presented along with the graphical representation of the connecting paths. The problem is formatted in a 50*50 grid space.

Table 3 : Experimental Problem-Set: Parameters Under Constrained Facility Location Situations.

Existing Facility Serial Number	Coordinate Locations of Existing Facilities	Interacting Weight with respect to New Facility
1	(47, 4)	0.15
2	(50,33)	0.15
3	(45,47)	0.20
4	(6, 45)	0.20
5	(0, 1)	0.20
6	(30, 2)	0.10
Barrier Number	**Co-ordinate of Vertices of Quadrilateral Barrier**	
1	(10,39) ; (20,40) ; (20,42) ; (10,43)	
2	(10,5) ; (15,5) ; (15,11) ; (12,11)	
3	(42,5) ; (46,5) ; (45,11) ; (43,11)	
4	(28,7) ; (31,4) ; (34,7) ; (31,10)	
5	(40,42) ; (42,37) ; (46,42) ; (43,44)	

Computerised Output of Analytical Software (with reference coordinate point for initiating optimality search iteration at 25, 25 chosen arbitrarily)Value : 29.43
Point-1 : 25 26 Value : 29.46
Point-2 : 26 25 Value : 29.34 (Next Reference Point with Minimum Value)
Point-3 : 25 24 Value : 29.42
Point-4 : 24 25 Value : 29.54

Graphical representation of the constrained inter-facility connecting path at optimality condition for the problem set is depicted in Figure2 .

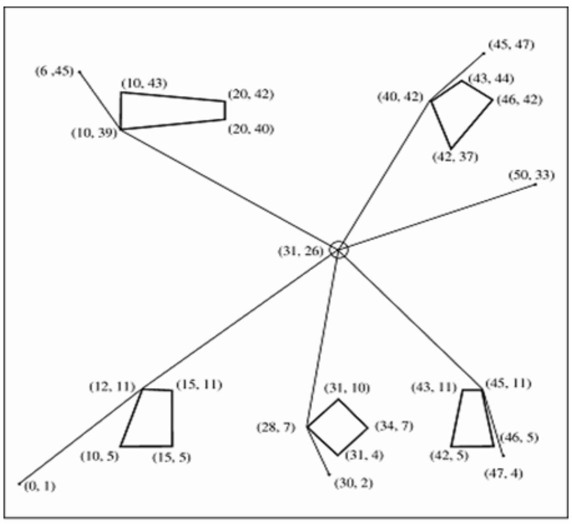

Figure 2: Graphical Representation of the constrained inter-facility connecting path at optimality condition.

Summary of Results:

	Coordinate	Value
1	26 25	29.34
2	27 25	29.27
3	28 25	29.20
4	29 25	29.15
5	30 25	29.12
6	30 26	29.11
7	31 26	29.10
8	31 25	29.11
Optimal: 7th	31 26	29.10

6. Conclusion

The validity testing of the new computational software is established through comparing the results in section 5 with the one using manual computation. With manual computation, it provides the very same value as has been obtained through the DANSORK software for the constrained problem condition as illustrated in the referred section. It has also been shown that the approximation with equivalent configuration does not affect the optimality result in any significant way. These establish the deploy ability of this software and the new obstacle search algorithm for both unconstrained as well as constrained conditions in facility location.

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