JJMIE

Volume 4, Number 3, June, 2010
ISSN 1995-6665
Pages 418 - 423

Jordan Journal of Mechanical and Industrial Engineering

A Mathematical Study for Investigation the Problems of Soft Shells Materials

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Abstract

The current study investigates the problems of soft shells theory for manufacturing composite products by superimposition on each 2D layers. Analytical and numerical methods are considered to study the layers imposing winding around the half-finished materials or pulling some additional shells on the surface of materials which are partly made. Based on this, the smoothness of layers, and the criteria of the absence of wrinkles and folds are obtained. Methods for calculation the deformations and residual stresses of the textile structure of used materials were established. The results obtained by the analytical and numerical methods indicated that it is possible to establish mathematical equations which can be applied to find the strains and stresses developed in the shells and bands and their pressure on the surfaces of covered solid. The results of the present work can also be implemented for manufacturing composite materials having complex geometric forms.

Key words: Soft Shell, Band; Deformation; Numerical Method;Strain and Stress.

1. Introduction

There are various processes to manufacturing the composite products by successive superimposition on each 2D layers that are glued mutually by some adhesive substance [1, 2]. In this study the problems of the capacity bands of the layer and shells that slightly resist bending forces (soft shells or membranes) are investigated. The production of these layers imposes on the surface of the materials during the forming process may be winding around the half-finished materials or pulling some additional shells on the surface of mistrials which are partly made. The composites may also be produced by successive inflating soft shells inside of some half-finished materials.

Whatever the methods are considered, some common problems of the theory of soft shells are appeared. Some of these problems are investigated in the present work, those that concern the changes undergone by textile structures of the bands or the shells as these bands and shells are fastened to the surfaces of the materials during the production stage . To predict the changes, it is necessary to develop methods for investigations of stress and strain distributions in the soft shells [2, 3]. The methods based on the mathematical study of stress-strain conditions of the soft shells [4] will be discussed in this paper. These studies include static equilibrium equations of the theory of the soft shell, the deformation equations of 2D materials as well as boundary conditions. The boundary value problems of various technologies for producing the composites are considered in connection with these equations. Solutions of these problems can be obtained by asymptotic analytical and numerical methods. Methods of small segment are also investigated when the problems of winding 2D bands around the solids of revolution which only in small measure differ from cylindrical ones. The bands of various textile structures and width are considered at different conditions of supplying band by feeding devises which determine the forms of cross sections of bands at different distances of the section to the solid surface.

Numerical methods are developed for investigation the interaction of the soft shells when they cover a solid. For calculations, the finite element method was used. By assuming that the strain energy density of the shell material is known, function of the shell strain measured in this method leads to minimization the potential energy of the whole shell. Problems of the minimization were reduced to the solution of non-linear algebraic equations sets. A certain methods with respect to the loading parameter are used to find the solutions of such equations.

2. Equilibrium Equations of the Theory of the Soft Shell

To derive the equilibrium equation two cylindrical shapes are suggested. The shape of R, φ, ζ is used to describe the form of the shell (see Figure 1) which is not subjected to any stress. This form can be defined by the equation:

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$$R = R_0(\varphi, \zeta) \quad (1)$$

The variables φ and ζ are considered as the Lagrange coordinates on the shell. In the other (spatial) system of cylindrical coordinates R, ψ, z (see Figure2) the form of the stressed shell is defined by the equation:

$$r = r(\psi, z) \quad (2)$$

where the axis z coincides with the axis ζ, r = the radius of the feeding cylinder

According to [5,6], the stress components σ_{11},

$\sigma_{12} = \sigma_{21}$ and σ_{22} are defined by the formulas

$$\boldsymbol{\sigma}_1 = \mathbf{e}_1\sigma_{11} + \mathbf{e}_2\sigma_{12} \quad (3)$$

$$\boldsymbol{\sigma}_2 = \mathbf{e}_1\sigma_{21} + \mathbf{e}_2\sigma_{22} \quad (4)$$

$$\mathbf{e}_1 = \frac{\partial r}{\partial \varphi} / \left|\frac{\partial r}{\partial \varphi}\right| \quad (5)$$

$$\mathbf{e}_2 = \frac{\partial r}{\partial \zeta} / \left|\frac{\partial r}{\partial \zeta}\right| \quad (6)$$

Where, $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_2$ are the stress vectors on coordinate lines $\zeta = const$ and $\varphi = const$, respectively.

in this case the equilibrium equation for the shell is taken in the vector form [7]:

$$\frac{\partial}{\partial \varphi}\left(\boldsymbol{\sigma}_1\left|\frac{\partial \mathbf{r}}{\partial \zeta}\right|\right) + \frac{\partial}{\partial \zeta}\left(\boldsymbol{\sigma}_2\left|\frac{\partial \mathbf{r}}{\partial \varphi}\right|\right) + \mathbf{q}\sqrt{g_{11}^0 g_{22}^0 - \left(g_{12}^0\right)^2} = 0 \quad (7)$$

Where the vector $\mathbf{q}$ is the intensity of the external forces acting on the shells surface and g_{ij}^0 and g_{ij} (i,j=1,2) are metric coefficients of the shell corresponding to unstressed and stressed state respectively.

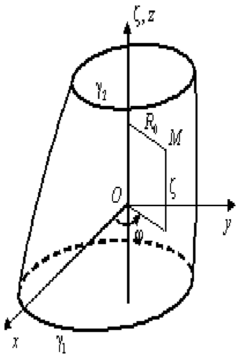

Figure 1.The shell in unstressed state

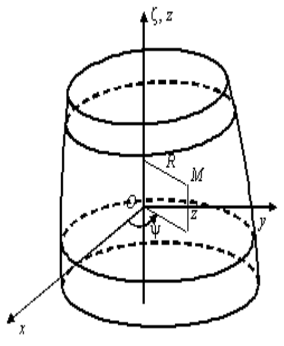

Figure 2. The shell on the solid's surface

3. Established equations of shell material

By considering the macroscopically measures of the shell deformation, the magnitude of the extension strains of coordinate lines are:

$$\varepsilon_1 = \lambda_1 - 1 \quad (8)$$

$$\varepsilon_2 = \lambda_2 - 1 \quad (9)$$

Where λ_1 and λ_2 are the elongation rates of coordinate lines.

The magnitude of the angle between these lines χ is determined by the equation:

$$g_{12} = \sqrt{g_{11}^0 g_{22}^0}\lambda_1\lambda_2 \sin\chi \cdot \quad (10)$$

λ_1 and λ_2 can be determined by formulas:

$$\lambda_1 = \sqrt{\frac{g_{11}}{g_{11}^0}} \quad (11)$$

$$\lambda_2 = \sqrt{\frac{g_{22}}{g_{22}^0}} \quad (12)$$

The established equations are obtained for deformation shells which allow the introduction of the potential strain energy. In previous works [5, 9], the equations are given in the following form:

$$\sigma_{11} = \frac{1}{\lambda_2\sqrt{g_{11}^0 g_{22}^0}}\left(\frac{\partial u}{\partial \lambda_1} + \frac{ctg\chi}{\lambda_1}\frac{\partial u}{\partial \chi}\right) \quad (13)$$

$$\sigma_{22} = \frac{1}{\lambda_1\sqrt{g_{11}^0 g_{22}^0}}\left(\frac{\partial u}{\partial \lambda_2} + \frac{ctg\chi}{\lambda_2}\frac{\partial u}{\partial \chi}\right) \quad (14)$$

$$\sigma_{12} = \sigma_{21} = -\frac{1}{\lambda_1\lambda_2 \sin\chi\sqrt{g_{11}^0 g_{22}^0}}\frac{\partial u}{\partial \chi} \quad (15)$$

Where u is the density of the strain energy, related to the unit of the area of the unstressed shell.

In [5] u was calculated as a function of $\chi, \varepsilon_1, \varepsilon_2$ for some various textile structures. This study will concentrate by considering the shell is made of nets with rectangular meshes.

4. Investigation of The Variation Principle of Pulled Shell on The Solid Surface

The surface of the solid on which the shell will be pulled (see Figure 2), can be defined by the following equation:

$$R = R(\mu, \psi, z) \quad (16)$$

where, μ is an artificially introduced parameter variation which leads to the changing of the solids form.

The radius-vector of any particle M of the shell when it is pulled on the solid surface can be presented as follows:

$$\mathbf{r} = \left(R_0(\varphi,\zeta) + \rho(\mu,\varphi,\zeta)\right)\cos\left(\varphi + \theta(\mu,\varphi,\zeta)\right)\mathbf{i} + \left(R_0(\varphi,\zeta) + \rho(\mu,\varphi,\zeta)\right)\sin\left(\varphi + \theta(\mu,\varphi,\zeta)\right)\mathbf{j} + \left(\zeta + w(\mu,\varphi,\zeta)\right)\mathbf{k} \quad (17)$$

where $\theta(\mu,\varphi,\zeta)$ is the increment of the angular coordinate of the particle M, $\rho(\mu,\varphi,\zeta)$ and $w(\mu,\varphi,\zeta)$ are the radial and vertical displacements, respectively; $\mathbf{i}, \mathbf{j}, \mathbf{k}$ are unit vectors along the axes x, y and z respectively.

It is obvious that the following equations are true:

$$R\left(\mu, \varphi + \theta(\mu,\varphi,\zeta), \zeta + w(\mu,\varphi,\zeta)\right) = R_0(\varphi,\zeta) + \rho(\mu,\varphi,\zeta) \quad (18)$$

$$\psi = \varphi + \theta(\mu,\varphi,\zeta) \quad (19)$$

$$z = \zeta + w(\mu,\varphi,\zeta) \quad (20)$$

The equation (18) shows that to obtain the full description of shell deformation it is enough to know the functions $\theta(\mu,\varphi,\zeta)$ and $w(\mu,\varphi,\zeta)$.

To describe the deformation of the shell on the surface of the solid, the expressions for the metric coefficients can be written firstly, to initial unstressed state, and, secondly, to final deformed state. According to equations (1-6), these coefficients are given by the following equations:

$$g_{11}^0 = \left(\frac{\partial R_0}{\partial \varphi}\right)^2 + R_0^2 \quad (21)$$

$$g_{22}^0 = \left(\frac{\partial R_0}{\partial \zeta}\right)^2 + 1 \quad (22)$$

$$g_{12}^0 = g_{21}^0 = \frac{\partial R_0}{\partial \varphi}\frac{\partial R_0}{\partial \zeta} \quad (23)$$

$$g_{11} = \left(\frac{\partial \mathbf{r}}{\partial \varphi}\right)^2 \quad (24)$$

$$g_{22} = \left(\frac{\partial \mathbf{r}}{\partial \zeta}\right)^2 \quad (25)$$

$$g_{12} = g_{21} = \frac{\partial \mathbf{r}}{\partial \varphi}\frac{\partial \mathbf{r}}{\partial \zeta} \quad (26)$$

Where, $\mathbf{r}$ is defined by Eq. (17) and Eq. (18) which allow to eliminate $\rho(\mu,\varphi,\zeta)$.

By the computation, it was assumed that all points of the bottom edge γ_1 of the soft shell have coordinates $\zeta = 0$, and points of the top edge γ_2 – have coordinates $\zeta = H$. Thus the potential energy of the whole shell equals

$$U = \int_0^H \int_0^{2\pi} u\sqrt{g_{11}^0 g_{22}^0 - g_{12}^0}\, d\varphi d\zeta \quad (27)$$

Where, u is a function of Φ, depending on displacements $\theta(\mu,\varphi,\zeta)$ and $w(\mu,\varphi,\zeta)$ of the shell particles and the first derivatives of these displacements:

$$u = \Phi\left(\mu, \varphi, \zeta, \theta, w, \frac{\partial\theta}{\partial\varphi}, \frac{\partial\theta}{\partial\zeta}, \frac{\partial w}{\partial\varphi}, \frac{\partial w}{\partial\zeta}\right) \quad (28)$$

to investigate the displacements $\theta(\mu,\varphi,\zeta)$ and $w(\mu,\varphi,\zeta)$, which satisfy the boundary conditions of the clamped edges, the conditions are given by the following equations:

$$\theta(\mu,\varphi,0) = \theta_0(\varphi) + \mu\theta_0^*(\varphi) \quad (29)$$

$$\theta(\mu,\varphi,H) = \theta_1(\varphi) + \mu\theta_1^*(\varphi) \quad (30)$$

$$w(\mu,\varphi,0) = w_0(\varphi) + \mu w_0^*(\varphi) \quad (31)$$

$$w(\mu,\varphi,H) = w_1(\varphi) + \mu w_1^*(\varphi) \quad (32)$$

where, θ_0, θ_0^*, θ_1, θ_1^*, w_0, w_0^*, w_1 and w_1^* are arbitrary given functions.

The analysis of the deformed state of the shell is based on the variation principle of the minimum of the potential strain energy [8]. Based on this principle, the equilibrium state of the shell corresponds to the minimum of the energy

5. Finite Element Method for the Investigation of Pulled Shell on the Solids' Surface

Minimization of the functional U was accomplished by the finite element method and small parameter method [9,10]. To apply the finite element method, the definition domain of functions $\theta(\mu,\varphi,\zeta)$ and $w(\mu,\varphi,\zeta)$ is subjected to the triangulation. These functions are approximated as follows [4]:

$$\theta = \sum_k T_k p_k \quad (33)$$

$$w = \sum_k V_k p_k \quad (34)$$

Where, p_k is shape functions, T_k and V_k are nodal magnitudes of $\theta(\mu,\varphi,\zeta)$ and $w(\mu,\varphi,\zeta)$ respectively, k =1,…, and r is the node number (number of coefficients X_k).

When using such approximations of U by the formula (14), U can be appeared as a function of coefficients T_k and V_k. By equating the partial derivatives of this function with respect to mentioned coefficients to zero, a group of algebraic equations is obtained. The solution provides the nodal displacements.

At the topmost and bottommost strings of nodes (their quantities will be denoted by $2m$) in accordance to Eqs. (21-26). The values of T_i and V_i may be given arbitrary. Thus, the following designations can be given:

$$X_{2k-1} = T_{k+m},\ X_{2k} = V_{k+m} \quad (35)$$

In this case U transforms into the function of X_k.

To derive equations for the calculation of the coefficients X_k, the conditions of minimum energy U can be represented by the following equation:

$$\frac{\partial U}{\partial X_k} = 0 \quad (36)$$

In order to analyze the results of nonlinear equation, the differentiation can be taken for all equations with respect to the parameter μ. As a result, a linear equation in regarding to derivatives $\frac{dX_k}{d\mu}$ can be obtained.

The last equation set may be represented in the following form:

$$C(\mu, X(\mu))\frac{dX}{d\mu} = B(\mu, X(\mu)) \quad (37)$$

Where, C is a matrix of the format $r \times r$, B is a vector of length r, X is a required vector of the length. Thus, the equation (37) represents a group of the ordinary differential equations from which $T_k(\mu)$ and $V_k(\mu)$ should be found. The solution of this group leads to the Cauchy problem if $T_k(0)$ and $V_k(0)$, or more definitely $X_k(0)$, are known values. By rewriting Eq. (37) in the form of:

$$\frac{dX}{d\mu} = C^{-1}(\mu, X(\mu))B(\mu, X(\mu)) \quad (38)$$

the required solution can be achieved by the following method:

$$\left.\begin{aligned} &X(\Delta\mu) = X(0) + \left.\frac{dX}{d\mu}\right|_{\mu=0} \\ &\Delta\mu = X(0) + C^{-1}(0, X(0))B(0, X(0))\Delta\mu \end{aligned}\right\} \quad (39)$$

$$\left.\begin{aligned} &X(\mu_k + \Delta\mu) = X(\mu_k) + \left.\frac{dX}{d\mu}\right|_{\mu=\mu_k} \\ &\Delta\mu = X(\mu_k) + C^{-1}(\mu_k, X(\mu_k))B(\mu_k, X(\mu_k))\Delta\mu \end{aligned}\right\} \quad (40)$$

Thus, the deformation of the shell can be calculated at any given value of μ. If the solution of this problem is known at $\mu = 0$, i.e. if initial conditions are known for Cauchy problem, this can be related to equation (38).

It is possible to specify various methods to establish such initial conditions. The elementary method consists one-parametrical group of solids with the parameter μ. There is an obvious trivial solution at $X = 0$, and if the group remains the same at $\mu = \mu_*$, therefore, the surface of a body will have the demanded form, and at $\mu = 0$ this surface coincides with a surface of no deformed shell.

By considering the position of a cylindrical shell with the radius R_0 and length H on the solid surface, mathematically, this can be represented by the following equation [11]:

$$R(z) = R_0 + \mu z + A\mu\sin\psi \quad (41)$$

Where, μ is the above mentioned parameter and A is some arbitrary chosen parameter.

In this case of zero value of the parameter μ it is possible to subject to such position of the shell on the solid surface, then this shell would not be deformed. This exact solution can be used as initial condition in the investigation of Cauchy problem for calculation of the state of the shell. Boundary conditions will be chosen so that one edge of the shell corresponds to the coordinate $z = 0$, and another to the coordinate $z = H$. In this case metric coefficients corresponding to the unstressed state of the shell are as follows:

$$g_{11}^0 = R_0^2,\ g_{12}^0 = g_{21}^0 = 0,\ g_{22}^0 = 1 \quad (42)$$

By referring to equations (13) and (23) the metric coefficients corresponding to the deformed state of the shell are given by:

$$g_{11} = \mu^2\left(A\cos\psi\left(1 + \frac{\partial\theta}{\partial\varphi}\right) + \frac{\partial w}{\partial\varphi}\right)^2 + (R_0 + \mu z + A\mu\sin\psi)^2\left(1 + \frac{\partial\theta}{\partial\varphi}\right)^2 + \left(\frac{\partial w}{\partial\varphi}\right)^2 \quad (43)$$

$$g_{22} = \mu^2\left(A\cos\psi\frac{\partial\theta}{\partial\zeta} + \left(1 + \frac{\partial w}{\partial\zeta}\right)\right)^2 + (R_0 + \mu z + A\mu\sin\psi)^2\left(\frac{\partial\theta}{\partial\zeta}\right)^2 + \left(1 + \frac{\partial w}{\partial\zeta}\right)^2 \quad (44)$$

If $\mu = 0$ then it is easy to notice that equations (42), (43) and (44) coincide

6. Analytical Method of Small Segment for Investigation of Shell Pulled on the Solid Surface

There are many difficulties in interpretation of above mentioned cases when numerical methods fail. As a rule, it is insignificant modification of boundary conditions to reach smoothness of shell and destroy convergence of computation process. For this reason the applying of analytical methods may give substantial contribution in development of numerical methods. Thus, the analytical investigations can be used under the assumption that the shell is made of linearly elastic net with rectangular infinitely small cells. In this case and referring to Eq. (27), the potential energy functional U can be written in the form:

$$U = \int_0^H\int_0^{2\pi}\left(k_1\varepsilon_1^2 + k_2\varepsilon_2^2\right)\sqrt{g_{11}^0 g_{22}^0 - g_{12}^0}\,d\varphi\, d\zeta \quad (45)$$

Where, k_1 and k_2 the coefficients describing elastic properties of the threads that are disposed along coordinate lines and confine meshes of the net.

The limitation of the analysis by axisymmetric problems of interaction between cylindrical shell that has radius R_0 and the solid obtained by the rotation around axis z of line given by the following equation:

$$R(z) = R_0 + \Delta + R_0\mu\cos\frac{\pi z}{2l} \quad (46)$$

where, Δ and l are arbitrary chosen quantities.

Because of axial symmetry the shell angular displacements $\theta(\mu, \varphi, \zeta)$ are zero and all the deformations are defined by axial displacements w, which do not depend on φ, and may be presented as follows:

$$w(\mu, \zeta) = u_0(\zeta) + \mu u(\zeta) + \ldots. \quad (47)$$

At zero and first order approximation of Euler's equation for extreme problem concerning U can be given in the following form [6]:

$$\frac{d^2u_0}{d\zeta^2}=0,\quad \frac{d^2u_1}{d\zeta^2}+\frac{k_2}{k_1}\frac{\pi}{2l}\frac{\Delta}{R_0}\sin\frac{\pi(\zeta+u_0(\zeta))}{2l}=0,\quad .... \qquad (48)$$

The solutions of various boundary values, problems for equations (48) can be obtained easily. For example, the formula for calculation of pressure exerted by shell with free edges on the solid surface can be represented by:

$$q=-\frac{k_2}{R_0}\left(\frac{\Delta}{R_0}\pm\mu\cos\frac{\pi\zeta}{2l}\right) \qquad (49)$$

7. Influence of the Deformation Properties of the Textile Materials

The influence of the deformation properties of the textile materials that are used during the production of composite products is certainly significant but not easy to be taken into account. Now, consider the deformations of structures of reinforcing textile tapes for manufacturing of composite products by winding process, the production of such composite materials shows increased requirements to uniformity of structures of tapes. Heterogeneity of the structures can be developed due to the deformations of winding process. By assuming that the tape has a structure of a plain weave and its basic strings go along a tape and have the visco-elastic properties. The deformations of these strings during winding process can be studied with reference to Figure3. From this figure, V_C is the vertical velocity of the cylinder on which the tape is reeled up, Ω and R are the angular velocity and the radius of the cylinder, ω and r are the angular velocity and the radius of the feeding cylinder; φ is the angle of geodetic lines along which the basic strings are settled down, L is the length of the bottom string part which is located between the cylinders.

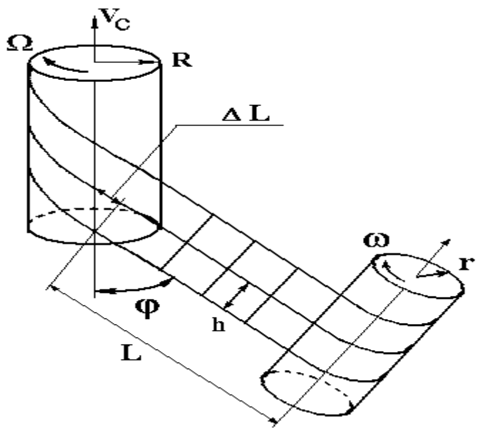

Figure 3. The winding of the tape

By considering that the axis of feeding cylinder is perpendicular to these strings, the following equation for calculation of the tension of string can be applied:

$$T = E\varepsilon_A + \frac{\mu}{L+\Delta L}(1+\varepsilon_A)(V_B - V_A) \qquad (50)$$

Where ε_A deformation of basic strings on the feeding cylinder:

$$V_A = r\omega,\quad V_B = R\Omega / \cos\varphi \qquad (51)$$

By referring to Eq. (50) the tension of strings are non-uniform and depends on their place in a tape and the elimination of this non-uniformity is possible by various methods.

8. Conclusion

1. A mathematical study of the stress-strain curve (behavior) based on the static equilibrium and established equations of membranes were developed to evaluate the problems of the manufacturing of composite materials.
2. The study considered that the little capacity of the layers assumed to be membranes with a small resistance to the bending moments.
3. To predict the changes in the textile structure of the layer materials, the stress-strain behavior of the layers (membranes or soft shells) was investigated.
4. By asymptotic analytical methods of small segment, the strain energy and finite elements methods were used to evaluate the boundary problems and textile structural changes of the layers.
5. A criterion of smoothness of layers was developed and a method of calculating the deformations of the textile structure of the layers was proposed.

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